# A SHORT COURSE

IN

# GRAPHIC STATICS

## FOR STUDENTS OF
## MECHANICAL ENGINEERING

BY

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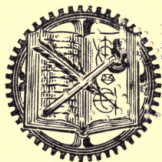NAVAL ARCHITECTS AND MARINE ENGINEERS, THE FRANKLIN
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*58 ILLUSTRATIONS*

# PREFACE

THE purpose of this book is to provide students of mechanical engineering with a brief course in Graphic Statics which will serve when the time to be devoted to this subject is short. Owing to the necessary limitations as to size, the treatment has been restricted mainly to the properties and general uses of the force and equilibrium polygons, these polygons being sufficient for the solution of most of the problems met in practice by mechanical engineers. While the design of trusses is, in general, the duty of the civil engineer, some attention has been given this subject, since such constructions as the Warren girder for an overhead crane, the walking beam of an engine, etc., fall under this classification. Examples, in full detail, have been included, so far as space would admit, since one good example is often of more service in instruction than many pages of theoretical investigation.

The discussion of principles, as given in this book, is largely a summary of similar portions of the authors' creatise, "The Elements of Graphic Statics," although there have been some minor additions. With the latter and the examples, the new material comprises nearly two-thirds of the text.

NEW YORK, October 1, 1911.

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# GRAPHIC STATICS

## CHAPTER I

### FORCE AND EQUILIBRIUM POLYGONS

The Force Triangle, its extension, the Force Poly-
gon, and the Equilibrium or Funicular Polygon form
the foundations of all important applications in practice
of the science of Graphic Statics. Consider first:

**1. Force Triangle.** In Fig. 1, the weight $W = 300$
pounds and the
weights $W_1$ and
$W_2$ are con-
nected at $O$ by
cords, two of
which pass over
frictionless sup-
ports at $b$ and $c$.
The system is
in equilibrium
and $W$ is at rest,
being held from
vertical or hori-
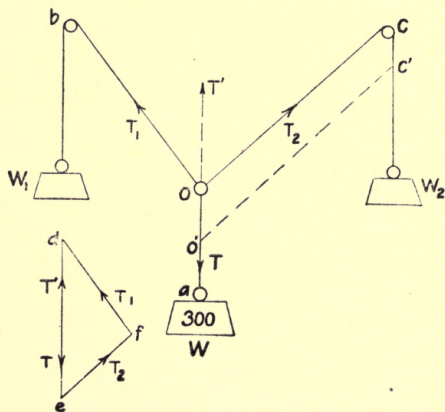


FIG. 1

zontal movement by the tensile stresses $T_1$ and $T_2$ in
the supporting cords, which stresses are equal to their

corresponding weights $W_1$ and $W_2$ respectively.    It is required to find the magnitudes of these weights.

On any scale — say, 200 pounds to the inch — lay off the line $de = T = W = 300$ pounds, parallel to the line of action $Oa$ of the stress or force $T$.    Draw $df$ and $ef$, parallel respectively to $Ob$ and $Oc$.    Then, measured by the same scale, $fd = T_1 = W_1 = 224$ pounds and $ef = T_2 = W_2 = 184$ pounds.

The triangle $def$ is the *force triangle* for the forces $T$, $T_1$, and $T_2$.    The direction of these forces is indicated by the arrowheads.    It will be seen that, in passing around the force triangle, the three forces have the same direction, which is the case always when the system is in equilibrium.    On the other hand, the resultant of $T_1$ and $T_2$ is a force $T' = T$, applied at $O$, and this would be shown by the line $ed$, whose direction, as given by the upper arrowhead, is the reverse of those of $T_1$ and $T_2$, in passing around the triangle.    Similarly, a force directed from $d$ to $f$ would be the resultant of $T$ and $T_2$, and one from $f$ to $e$, the resultant of $T$ and $T_1$.    These principles are general, *i.e.*, *if, in any force triangle, the forces have the same direction in passing around the triangle, the system is in equilibrium ; and, second, the force represented by any side of a force triangle is the resultant of the forces represented by the other two sides, if its direction, in passing around the triangle, is the reverse of theirs*.    The system, which consists of two forces and their resultant, is not then in equilibrium, since a resultant, unbalanced force exists.

**2. Force Polygon.**   The principles governing the con-
struction of the force polygon are an extension of those
of the force triangle.   Thus, Fig. 2, let there be four
forces $T_1 \cdots T_4$ acting at the angles shown, the mag-
nitude of each being
100 pounds, and all
meeting at the point
$O$ to support the
weight $W$, the mag-
nitude of which is
required.

On a scale of 100
pounds to the inch,
lay off $ab$, $bc$, $cd$, and
$de$, parallel respec-
tively to the lines of
action of $T_1$, $T_2$, $T_3$,
and $T_4$.   Then, $ae =$
$T' = 270$  pounds  is
the  resultant of the
four forces, and is
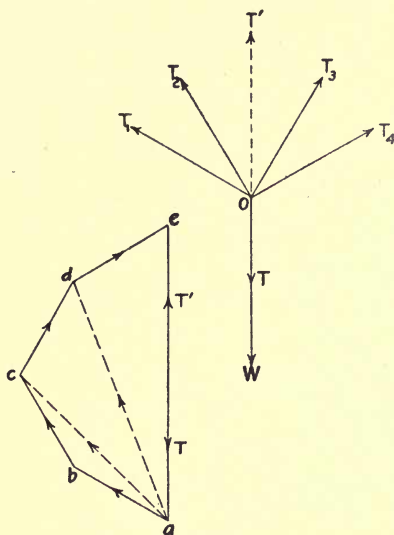


FIG. 2

therefore equal in magnitude to $W$ and opposite in
direction to $T$, the stress produced by $W$.   This follows
since, if the polygon be divided into triangles by the
dotted diagonals shown, $ac$ is the resultant of $ab$
and $bc$, $ad$ is the resultant of $ac$ and $cd$, and $ae = T'$
is the resultant of $ad$ and $de$ and therefore of the four
forces.   The principles established for the force triangle
apply also to the force polygon.   With either the tri-
angle or the polygon, the sides representing forces may

be drawn in any order, the result being always the same, since *the force polygon is essentially but the graphic addition of forces*, *i.e.*, the horizontal and vertical components of the resultant of a system of forces are, respectively, the algebraic sum of the horizontal and vertical components of the forces forming the system.

The forces considered in Figs. 1 and 2 are *concurrent*, *i.e.*, their lines of action meet at a common point *O*. With *non-concurrent forces*, the lines of action may be either inclined to each other or parallel, but in neither case have they a common point of intersection. The force polygon gives only the magnitude of a force and the inclination of its line of action ; it does not fix the location of that line within the body or area considered. For these reasons, such polygons can determine the equilibrium of concurrent forces only, the location of whose lines of action is governed by the fact that they must all intersect at one point. With non-concurrent forces, the same force polygon may be drawn for forces of the same magnitude and inclination, when these forces are, and are not, in equilibrium. Thus, Fig. 1, if the force $T_2$ be transferred to the new line of action $O'c'$, the system becomes non-concurrent, is no longer in equilibrium, and yet the force triangle *edf* remains the same.

**3. Equilibrium Polygon.** In order to determine the equilibrium of a system of non-concurrent forces, both the force and equilibrium (funicular) polygons must be drawn. Thus, let Fig. 3 represent non-concurrent forces

$P_1 \cdots P_5$ applied to a rigid body, the magnitude, direction, and line of action of each force being known and the system being in equilibrium. The force polygon for these forces is $a \cdots e$, as shown below. Starting at any point on any line of action, draw the equilibrium polygon $A \cdots E$, of any shape, the only conditions being that its sides $S_1 \cdots S_5$ shall be the same in number as the forces, and that each pair of sides shall intersect on the line of action between them. In the force polygon, draw the *rays* $S_1 \cdots S_5$ parallel to their corresponding
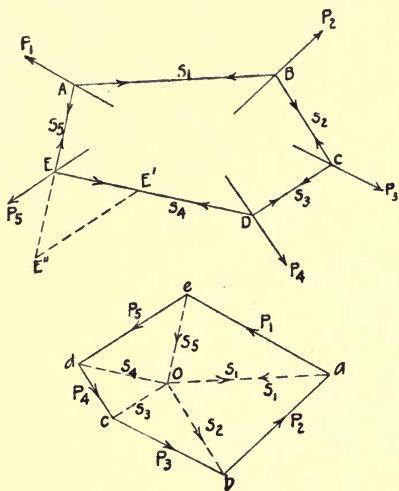


FIG. 3

ing sides in the equilibrium polygon. It will be found that these rays all intersect at a common point $O$, called the *pole*.

Now, consider the equilibrium polygon as a jointed frame, composed of rigid bars pivotally connected at the joints $A \cdots E$, this frame being in equilibrium under the action of the external forces $P_1 \cdots P_5$ and the stresses $S_1 \cdots S_5$ in the sides. Since the frame is in equilibrium, each joint must also be in equilibrium. Therefore, for joint $A$, the triangle $Oae$ is the force triangle for the

force $P_1$ and the stresses $S_1$ and $S_5$. The direction of $P_1$ is given, and this determines the direction of $S_1$ and $S_5$, with regard to joint $A$, as shown by the arrowheads in the force polygon, the three forces being in equilibrium and hence having the same direction in passing around the triangle. Transferring these directions to the equilibrium polygon, it will be seen that, since $S_1$ and $S_5$ both *act away from* joint $A$, they must both be tensile stresses.

The arrowheads just drawn on the rays relate only to the direction of the stresses with regard to joint $A$. Each joint must be treated separately in this respect. Thus, at joint $B$ the direction of $P_2$ is known and also that of $S_1$, since the latter has already been found to be tensile and therefore pulls away from joint $B$, reversing its direction in the force polygon as given for joint $A$. As the stress $S_2$ must have the same direction in the triangle *Oba*, it follows that it is tensile. $S_3$ and $S_4$ are also found to be tensile stresses. If any stress had *acted toward* any joint, it would have been compressive. The *character of a stress*, as to tension or compression, is thus given by its direction in the force polygon, when that direction is transferred to the line of action of the stress at the joint in question; its *magnitude* is shown by its length in the force polygon, as measured by the force-scale of the latter.

As stated, Fig. 3 represents a system of forces in equilibrium. That the equilibrium polygon must close for equilibrium to exist, may be shown by transferring the force $P_5$ to the new line of action $E'E''$. The

system will then be no longer in equilibrium, the side $DE$ will end at $E'$, the side $AE$ will be prolonged to $E''$, and the polygon will not close. In any force polygon, the rays all meet at the pole $O$, because each ray is common to two adjacent force triangles. It is evident, also, that an infinite number of equilibrium polygons can be drawn for the same system of external forces, since two consecutive rays simply show the magnitude and inclination of the two stresses which, at these angles of inclination, will hold the external force at the corresponding joint in equilibrium. If the inclination of the sides of the equilibrium polygon be changed, the magnitude of these stresses will be altered correspondingly. Hence, the pole may fall at any point within or without the force polygon ; or, reversing the operation, any point may first be selected as a pole, the rays drawn to the vertices of the force polygon, and then the equilibrium polygon constructed with its sides parallel to these rays. Finally, the equilibrium polygon may be substituted for any rigid body on which a system of forces — as $P_1 \ldots P_5$ — acts, since a force may be considered as applied at any point in its line of action.

When the lines of action of the forces are parallel, the same principles apply, except that the force polygon now becomes a straight line. Thus, let Fig. 4 represent a simple beam of 24 feet span, supported at $A$ and $B$, loaded with weights $P_1$ and $P_3$ of 300 and 200, pounds, respectively, and having a vertical, upward thrust, $P_2 = 100$ pounds. It is required to determine the magni-

tudes of the vertical reactions $R_1$ and $R_2$ at the left and right supports, respectively.

On the load-line $ad$ lay off, to any convenient scale, the downward force $P_1 = 300$ pounds $= ab$, the upward force $P_2 = 100$ pounds $= bc$, and the downward force $P_3 = 200$ pounds $= cd$. Take any point $O$ as the pole, and draw the rays $Oa$, $Ob$, $Oc$, and $Od$. Starting at any point $F$ on the line of action of $P_1$, draw the equilibrium polygon whose sides $FG$, $FH$, $HK$, and $KL$ are parallel, respectively, to the rays $Oa$, $Ob$, $Oc$, and $Od$. Finally, draw the closing side $LG$, and the corresponding ray $Oe$. Then, measured to the load-scale, $de = R_2 = 191.7$ pounds, $ea = R_1 = 208.3$ pounds, and the closed force polygon is the straight line $abcdea$.
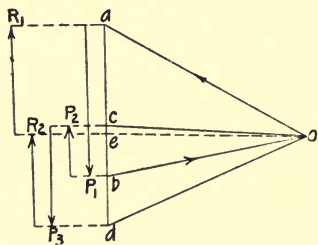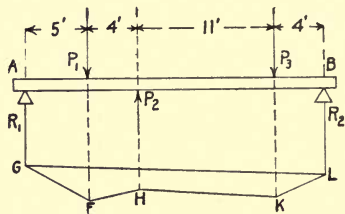


FIG. 4

It will be seen that, in selecting a pole and drawing the rays and the corresponding equilibrium polygon, we have simply assumed two stresses at each joint of the latter polygon which will hold the external force at that joint in equilibrium, the lines of action of the stresses being determined by the inclination of the rays. Thus, at joint $F$, the assumed stresses have

the lines of action *FG* and *FH* and their magnitudes are equal to *Oa* and *Ob*, respectively, when measured on the scale of the force polygon. Since the three forces $P_1$, *Oa*, and *Ob* have the same direction in passing around the triangle *Oab*, the system at joint *F* is in equilibrium. The same principles apply to each of the other joints. The direction of the closing line *LG* of the polygon determines that of the ray *Oe*, and hence the magnitudes of the two reactions, since the latter are, in effect, vertical, upward forces, held in equilibrium by the stress in *LG* or *Oe* acting with that in *FG* or *Oa* for $R_1$, and with that in *KL* or *Od* for $R_2$.

**4. Conditions of Equilibrium.**—The forces considered in graphical processes are *complanar*, *i.e.*, their lines of action all lie in the same plane. As to such forces, it will be seen from the foregoing that, for the equilibrium of:

(*a*) *Concurrent forces*, the force polygon must close; and, conversely, if the force polygon closes, the system is in equilibrium.

(*b*) *Non-concurrent forces*, both the force and equilibrium polygons must close; conversely, if the force polygon and any equilibrium polygon close, the system is in equilibrium.

In the graphical analysis of the forces acting on a moving body, the latter is assumed to be in momentary equilibrium under the action of the driving forces, the reactions at journals or other supports, and the forces

due to resistances to motion. The forces or stresses thus determined apply only to the given position of the moving parts.

When it is desired to use graphical methods with forces whose lines of action lie in different planes, the components of these forces in one common plane for all should be found. These components can then be treated as a complanar system.

<center>EXAMPLES</center>

In the following examples of the application of the force polygon, the friction of journals and other bearings is not considered. When a body rests on another, as a crosshead on its bearing, there is a reacting pressure from the bearing, equal and opposite to the downward pressure upon it. While the parts are at rest, this reaction is perpendicular to the surfaces in contact; when they move, the reaction would still be normal to these surfaces, if there were no friction (Art. 30); by friction, its line of action is diverted from the normal by the amount of the angle of friction, as will be explained later. The total reaction, which is the resultant of the indefinite number of indefinitely small reactions at all points of the bearing, is assumed theoretically to act at the centre of the bearing surface; in practice, this is only approximately true, since uneven wear makes the bearing pressure variable throughout.

1. BELL-CRANK. Taking the simplest example first, consider the bell-crank $AOB$, Fig. 5, journalled at $O$, driven by the link $CA$ with a force $P_1 = 100$ pounds,

and driving the link $BD$, whose resistance $P_2$ is to be determined.   The bell-crank is assumed to be at mid-stroke.

The crank is in momentary equilibrium under the action of the forces $P_1$, $P_2$ and the reacting pressure $R$ from the journal, which reaction is opposed to the resultant of the two forces. Since equilibrium exists and the forces are concurrent, the three lines of action must meet at a common point $E$, which is one point in the line of action of the reaction $R$.   The other point necessary is determined by the fact that, as friction is disre-



FIG. 5

garded, the reaction is normal to the contact-surfaces of the bearing, and hence its line of action passes through the centre $O$.   Therefore, in the force triangle $abc$, draw $ab = P_1 = 100$ pounds and lay off $bc$ and $ca$ parallel, respectively, to $EO$ and $BD$.   Then, on the same scale, $bc = R = 180$ pounds and $ca = P_2 = 150$ pounds.

2. PAWL AND RATCHET.   Figure 6 represents a link $AB$, driving with a force $P_1 = 50$ pounds the crank $BO$ loose on the shaft $O$ and carrying the pawl $C$, which engages the ratchet $D$ keyed to the shaft.   It is required to determine the resistance of, and the reaction upon, the pawl-crank.

The crank is in momentary equilibrium under the action of the driving force $P_1$, the backward thrust $P_2$, due to the resistance of the ratchet, and the reaction $R$ from the shaft, which reaction is equal and opposite to the resultant of $P_1$ and $P_2$.

Since equilibrium exists and the forces are concurrent, the lines of action of the latter meet
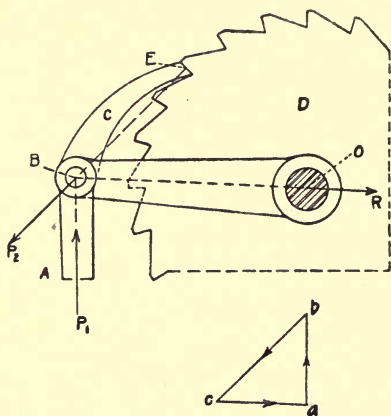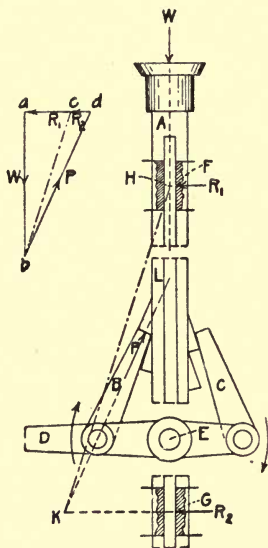


FIG. 6                    FIG. 7

at the common point $B$   To draw the force triangle, lay off $ab = P_1 = 50$ pounds, parallel to $AB$ and set off $bc$ and $ca$, parallel respectively to $EB$ and $BO$.   Then, $bc = P_2 = 70$ pounds and $ca = R = 50$ pounds.

3. RATCHET-RACK.   Figure 7 shows, in partial elevation, the double ratchet-rack, a mechanism sometimes used as a jack for raising light weights.   The rack $A$ has ratchet teeth on two opposite sides, which are alter-

nately engaged by the two oppositely inclined pawls, $B$ and $C$, mounted on the lever $D$, which is pivoted at $E$ on the framing (not shown). The rack is guided by two central ribs which reciprocate in guide-grooves formed in the frame. In Fig. 7, the driving lever is assumed to be at mid-stroke, with the pawl $B$ lifting the rack and the weight $W$ carried by the latter.

The thrust of the driving pawl is applied so near the centre line of the rack that its leverage and the consequent tendency to cant the rack are but slight. Hence, the reactions $R_1$ and $R_2$ of the bearings press toward the left and from the right sides of the grooves. These reactions are assumed to act from the respective centres, $F$ and $G$, of the bearing surfaces; the reaction shown at each bearing is taken as that on both ribs.

The rack is in momentary equilibrium under the action of the weight $W$, the two reactions, and the thrust $P$ of the driving pawl. There are thus four forces which are not concurrent, since the two reactions are parallel. The analysis of these forces can be made either by the use of the force and equilibrium polygons or by the method of resultants, as given below.

Equilibrium of this system of four forces can exist only when the resultant of any pair of them is equal to that of the other pair, and the two resultants are opposite in direction and have the same line of action. The lines of action of the weight $W$ and the resultant $R_1$ intersect at $H$, which is one point on the line of action of their resultant; similarly, the lines of action of $P$ and $R_2$ meet at $K$, which is a point on the line of action of

their resultant. Since the two resultants must have the same line of action, it is obvious that this line is $HK$.

Take $W$ as 300 pounds, and, in the force polygon, lay off $ab = W = 300$; draw $bc$ parallel to $HK$ and $ac$ parallel to $R_1$. Similarly, from $b$ and $c$ lay off $bd$ and $cd$, parallel respectively to $KL$ and $GK$. Then, $ac = R_1 = 100$ pounds, $cd = R_2 = 40$ pounds, and $bd = P = 334$ pounds. While this method would be similar for any distance between the bearings, and between the rack and the pin of the driving pawl, the numerical results, as above, apply only to the spacing shown in the figure and to the given position of the parts.

4. STATIONARY ENGINE. Let Fig. 8 represent diagrammatically the ordinary stationary steam engine, the
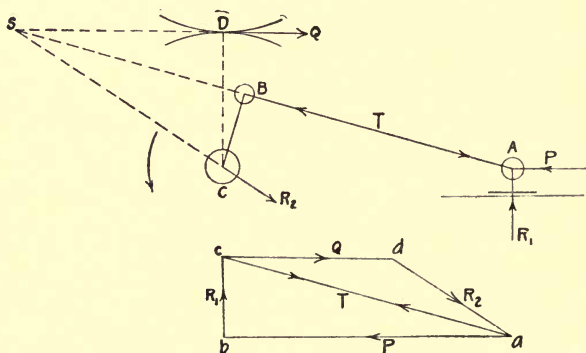


FIG. 8

piston pressure $P$ being applied at the crosshead pin $A$ and transferred through the connecting rod $AB$ to the crank $BC$ keyed to the shaft $C$, which rotates in an anti-clockwise direction.

First, to determine the reaction at the crosshead bearing: disregarding friction, this reaction will be perpendicular to the contact-surfaces and therefore vertical. The crosshead pin is thus in equilibrium under the action of three forces: the piston pressure $P$, acting horizontally and from right to left; the diagonal, downward resistance $T$ of the connecting rod; and the vertical and upward reaction $R_1$ of the crosshead bearing. Since equilibrium exists, the lines of action of these three forces intersect at a common point, and this point must be $A$, as $P$ and $T$ meet there. In the force polygon $abcd$ lay off, to any convenient scale, $ab = P$; draw $ac$ and $bc$, parallel respectively to the lines of action of $T$ and $R_1$. Then, on the same scale, $ca = T$ and $bc = R_1$.

Second, to find the reaction at the shaft bearing: to ascertain this, the character of the resistance must be assumed. Suppose that a spur gear $CD$ is keyed to the shaft, and that this gear meshes at $D$ with a similar gear which it drives. There will be evidently then at $D$ a resistance $Q$, acting from left to right and perpendicular to the radius $CD$. The conditions are thus similar to those of Example 1, since, as the gears and crank are keyed to the shaft, the three members virtually form a bell-crank which is pressed to the left at $B$ by the force $T$, now a driving force; is forced to the right by the resistance $Q$, acting at $D$; and is held from rising out of the bearing by the reaction $R_2$ of the cap of the latter. The lines of action of $Q$ and $T$ meet at $S$. Hence, for equilibrium, that of $R_2$ must be $SC$. In the force polygon, we have already found the magnitude of

the force $T$, which now acts from $a$ to $c$. Therefore, from $a$ lay off $ad$, and from $c$ lay off $cd$, parallel respectively to the lines of action of $R_2$ and $Q$. Then, $cd = Q$ and $da = R_2$.

5. PILLAR CRANE. In the crane shown diagrammatically in Fig. 9, $AB$ is the tie, $AC$ the boom, and $BC$ the
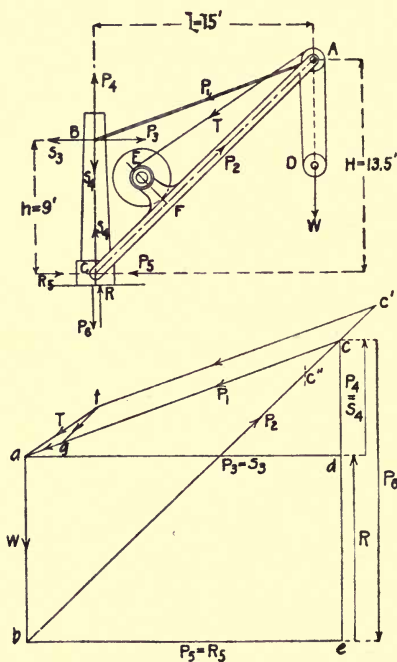


FIG. 9

post or pillar. The weight $W$ is suspended by a crane hook from the block $D$. One end of the hoisting rope is fastened to the tie-rod and the other passes around the sheaves in blocks $D$ and $A$ and thence to the hoist at $E$, the latter being secured to the boom. Let the blocks be single-sheaved with a velocity-ratio of 2 and a consequent tension $T = W/2$ in the hoisting rope. Take $W$ as 5 tons (10,000 pounds) and neglect the weight of the crane. It is required to determine the stresses in the tie, boom, and post, and to show the general effect on the two former stresses of the tension $T$.

First, neglect $T$, *i.e.*, assume $W$ to be simply hung from, and supported by, the joint at $A$. This joint is then in equilibrium under the action of $W$ and the loads $P_1$ and $P_2$ on the tie and boom, respectively. In the force polygon $abc$, lay off $ab = W = 5$ tons, and draw $ac$ and $bc$, parallel respectively to $AB$ and $AC$. Then, $ca = P_1 = 8.8$ tons is the tensile load on the tie, and $bc = P_2 = 11.4$ tons is the compressive load on the boom, due to the resultant along $cb$ of $W$ and $P_1$.

Joint $B$ is in equilibrium under the action of the tensile force $P_1$ and the stresses which its horizontal and vertical components produce. The horizontal component, $P_3 = ad = 8.3$ tons, acts to bend the pillar, and this bending is resisted by stresses whose resultant $S_3$ is equal and opposite to $P_3$ and acts at $B$. The vertical component, $P_4 = dc = 2.9$ tons, tends to stretch either the post or vertical ties between $B$ and $C$. This force is resisted by an equal and opposite tensile stress $S_4$.

At joint $C$ there are acting the diagonal thrust $P_2$ on the boom, the tensile stress $S_4$, an upward reaction $R = W$ on the collar- or pivot-bearing of the post, and a horizontal reaction $R_5$ from the bearing at the base. The horizontal and vertical components of $P_2$ are $P_5$ and $P_6$, respectively. The force polygon is $cbedc$, in which $cb = P_2$, $be = P_5 = P_3$, $ed = R = W$, $dc = S_4$, and $ce = P_6 = S_4 + R = 7.9$ tons.

Now, consider the tension $T$ in the hoisting rope between the joint $A$ and the hoist $E$. This tension has four direct and indirect effects: it acts as a tensile force, diagonal and to the left, on the joint $A$; as a result, its

c

component parallel to the tie takes a part of the original load on the latter, and its component parallel to the boom acts both as an additional compressive load between $A$ and the point of attachment $F$ of the hoist, and as a tensile load between $F$ and the lower end of the boom at $C$.

To lay out the new force polygon for joint $A$, draw $af = T = W/2$ parallel to $EA$, and from $f$ draw $fc'$ parallel to $AB$ and meeting $bc$ prolonged at $c'$. The polygon is then $abc'fa$ and $c'f = 7.5$ tons is the tensile load on the tie, while $bc' = 12.7$ tons is the compressive load on the boom between the points $F$ and $A$. It will be seen that these changes in the two loads amount simply to adding the component $gf$ of $T$ parallel to the boom to the compressive load on the latter, and to deducting the similar component $ga$ from the original tensile load on the tie. Now, from $c$ lay off $cc'' = fg$. Then, $c''b = 10.1$ tons is the new compressive load on the boom between $F$ and $C$. These changes in the load on the tie and in the thrust on the lower end of the boom alter correspondingly the values of $P_3$, $P_4$, $P_5$, and $P_6$, with their resulting stresses and reactions, although $R$ is still equal to $W$. The general effect of the tension $T$ is then to increase the thrust on the upper section of the boom and to decrease the tension in the tie.

It will be observed that the values which have been found are, in general, loads and not stresses. The stress in the tie is pure tension; the boom acts as a column, compressed between $C$ and $A$ and loaded at $C$ and $F$; the stresses in the post or pillar depend on

the method of connection of the tie and boom to each other and to the vertical member between them. In the ordinary pillar crane, vertical tie-rods, connecting the lower ends of the main tie and boom, take the vertical components of the stresses in these two members. The pillar is subjected to bending by the force $P_3$, and to compression from the direct vertical load $W$, and there is both a vertical reaction equal to $W$ on the pintle at the upper end of the pillar and a bending stress due to $P_3$. Hence, to determine the unit-stress in the metal, the conditions under which each member of the crane works must be considered.

In the foregoing the weight of the crane has been disregarded, and this weight must be taken into account for an accurate determination of the stresses. The weight of any member of any structure acts at the centre of gravity of that member, and is assumed to be divided between the two points of support of the latter in inverse proportion to the lengths of the two segments between these points and the centre of gravity. Thus, if a member be 10 feet long and weigh 1000 pounds, and if the centre of gravity be 7 feet from the left end, 300 pounds will be sustained by the support at that end and 700 pounds by the right support. Proceeding thus, we have a series of vertical forces, due to this weight or ' dead load,' acting at such joints as $A$, $B$, and $C$. For full accuracy, these vertical forces must be considered in drawing the force polygons. This method is general, and applies to all structures. If $W_1$ be the total weight of the crane, the pressure

on the bearing at the base of the pillar will be $W + W_1$ = $R'$, the corresponding vertical reaction there.

The weights of the members should first be estimated, in a tentative design, from the data of cranes previously built.   If the members as designed exceed these weights, the forces, stresses, and dimensions must be recalculated until a reasonable approximation to accuracy is reached.   As to pillar cranes in general, a reasonable estimate, as given by good practice, of the weight of all parts except the pillar and its immediate connections, is one-half of the maximum 'live load' to be carried by the crane hook ; the distance of the centre of gravity of these weights from the centre of the pillar is usually about one-fourth the radius or swing of the crane.



FIG. 10

6. SHEER LEGS.  The analysis of the stresses in sheer legs differs from the preceding examples, since the forces and stresses are not all in the same plane.

The legs in Fig. 10 are 130 feet long; the angle between them is 20 degrees; at the inclination shown, the plane passing through them makes an angle of 60 degrees with the horizontal, and one of 20 degrees with the tie or back leg. Taking the load as 100 tons, it is required to find the loads on the members. The weight of the structure will be neglected.

As stated, the lines of action of the loads and stresses lie in two planes: the vertical plane passing through the tie and the inclined plane in which the legs lie. As graphic methods are applied to forces in one plane only, we must find first the resultant, along the intersection of the two planes, of the forces in the inclined plane. The joint $O$ is in equilibrium under the action of this resultant, the load $W$, and the stress $T$ in the tie. In the force triangle $(A)$ lay off $ab = W = 100$ tons; draw $bc$ and $ac$ parallel, respectively, to the tie $ON$ and the legs $OM$. Then, $bc = T = 143$ tons is the load on the tie or back leg, and $ca = P = 220$ tons is the resultant, along the intersection of the two planes, of the compressive forces on the sheer legs.

To find the load or stress $P_1$ on each leg, the resultant $P$ must be resolved along the lines of the legs. Therefore, in $(B)$, lay off $ac = P$, and from $a$ and $c$ draw $ad$ and $cd$, each making an angle of $20°/2 = 10$ degrees with $ac$. Then $cd = da = P_1 = 112$ tons. The horizontal component of $da$ or $P/2$ is $ed = Q = 22.5$ tons, which is the outward force acting on each leg at $M$ to spread the pair apart.

At the joint $N$ the diagonal tensile force $T$ is resisted

by the vertical downward pull $R$ of the foundation bolts, and the horizontal tensile stress $T_1$ of the screw or other mechanism used for raising or lowering the legs. Hence, joint $N$ is in equilibrium under the action of the forces $T$, $R$, and $T_1$. In ($C$) lay off $cb = T$; draw $bf$ vertically from $b$; and, horizontally from $c$, draw $fc$. Then $bf = R = $ 91.5 tons, and $fc = T_1 = $ 110.5 tons.

In these diagrams the hoisting rope is assumed to act along the tie; if its line of action be inclined to the latter, the principles of Example 5 must be applied. With screw mechanism, the point $N$ moves to and fro horizontally in raising or lowering the sheer legs; if a tackle be substituted for the tie or back leg, the point $N$ should be fixed. Each of the sheer legs is compressed like a column by the force $P_1$ and its weight; the latter, acting at the centre of gravity of the leg, also produces bending stresses. The tie is in tension, but its weight acts similarly to produce bending. Owing to the lengths of these members, their weights are relatively large, and hence the effects of these combined stresses must be considered carefully in designing the parts.

# CHAPTER II

**5. Framed Structures.** The term 'framed structures' is used to designate such constructions as roof and bridge trusses, the lattice girders for overhead cranes, the walking beam of an engine, etc. In all such structures, the loads, however applied, are distributed among the members, which are thus simultaneously under strain. The fundamental requirement in truss design is that the members shall be so combined as to form a series of triangles whose sides shall be capable of withstanding compressive stress, if necessary. This requirement is due to the fact that a triangle — assuming rigid sides and hinged vertices — is the only polygon which will not change its shape when loads are applied at one or more vertices. The members forming a truss may be designed for tension only as in a *tie*, for compression only as in a *strut*, or for both stresses as in the *tie-strut*.

(*a*) *Assumptions*. In the graphic analysis of trusses, two basic assumptions are made : first, the loads, of whatever character, are considered as transferred to, and applied at the joints of the truss only, which would produce, only direct tension or compression in the members

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which meet at a joint; and, second, these loads are held
in equilibrium at each joint by the internal stresses act-
ing in the members united there.

In practice, the first of these conditions is met by so
designing the structure as to transfer the loads to the
joints; the *purlins* and *jack rafters* of a roof truss and
the *floor system*, or transverse and longitudinal beams
and sleepers, of a bridge truss serve this end.    It is
true that bending stresses, due to the weight of the
members, and, in some cases, to the uniform load carried
by them, do exist in the members, and when the latter
are relatively large, these stresses should be considered
in addition to the direct stresses for which the parts are
designed.    The second of the assumptions, as above,
must be true for the stability of the structure.

(*b*) *Loads*.   All trusses carry as a *dead load*, the weight
of the structure, which is assumed, in general, to be
divided proportionately and concentrated at the joints.
In roof trusses whose roof-surface is inclined at less
than 60 degrees, there is in winter an additional uniform
load in the weight of the snow on the roof, and in all
such trusses there is also an intermittent load, in the
wind pressure on one side or the other of the roof.
Bridge trusses have similar wind-loads, but their chief
burden is the *live* or *moving load* which crosses the
bridge.    These statements, as to dead and live loads,
apply also to the trusses for overhead cranes.

**6. Stress Diagrams.**   Since it is assumed that all
loads are concentrated at the joints, that the members

are subjected to direct axial stress only, and that each joint is in equilibrium under the action of the loads applied there and the stresses whose lines of action meet at that point, it is evident that the joints of a truss, such as is shown in the upper diagram, Fig. 11, are similar in principle to the joints of an equilibrium polygon. It should be understood that the truss (*force* or *space*) diagram is not such a polygon and that this similarity relates only to the joints and the elementary triangles of the truss.

Owing to this similarity, there can be drawn, for each joint, a force polygon, since the directions of all lines of action and the magnitudes of the external loads are known. Further, since each member connects two joints, one side of each force polygon is common to it and to the polygon immediately following. Hence, the series of force polygons, for a complete truss or other similar structure, is consecutive and forms the *stress diagram.*

The *character of the stress* in a member — either tensile ( + ) or compressive ( − ) — is indicated by the direction of the stress with regard to the joint, being tensile if it act away from the joint and compressive if it press toward the latter. This direction, to or from the joint, is given, as shown previously, by that of the stress in passing around the force polygon. In constructing this polygon, the loads and stresses about a joint are taken usually in clockwise order, and, starting at any joint, the remainder may be taken consecutively, or, as is sometimes necessary, alternately from the two sides of the truss.

The stresses in the members of a truss may also be determined by Rankine's "Method of Sections." This is an analytical process, the application of which is shown in Art. 22.

## EXAMPLES

7. ROOF TRUSS, DEAD LOAD. Figure 11 gives the skeleton diagram of a steel roof truss, having a *lower chord AA'*, an *upper chord* composed of *rafters* inclined at about 30 degrees to the horizontal, and connecting *diagonal members* or braces. The *span AA'* is 36 feet, the *rise* from chord to peak is 10 feet, each rafter is 20 feet 9 inches long, and the trusses are spaced 20 feet apart. The joints of the rafters divide the section of roof carried by each truss into *panels* of equal width, 1–2, 2–3, etc.; the length of the panels is 12 feet,
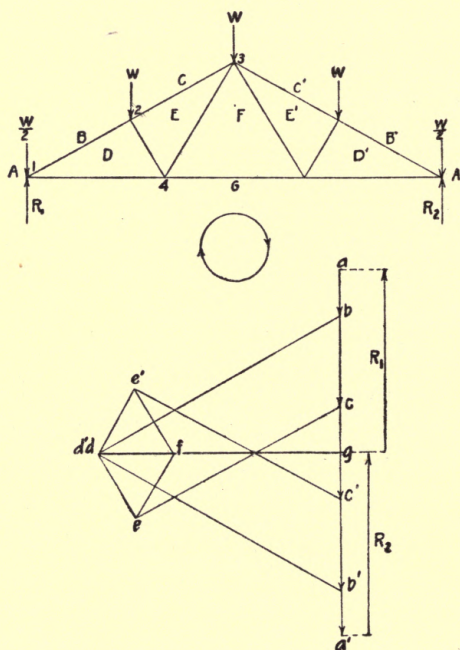


FIG. 11

*i.e.* the distance between the centre-lines of the adjacent spaces between the trusses. The total weight of the truss and roof covering is borne by the rafters, and each joint (*apex* or *panel-point*) of the latter is assumed to support one-half of the load on each of the adjacent panels and a corresponding share of the weight of the truss. In Fig. 11, there are thus three joints having a full panel-load, and one at each of the two supports, $A$ and $A'$, having one-half of that load. The total dead load is sustained at the supports by the two vertical reactions, $R_1$ and $R_2$. With dead and snow loads, these reactions are equal for a symmetrical truss.

The *dead load* consists of the weight of the truss and that of the roof covering. In designing, the weight of the truss may be estimated from those of trusses previously built, or an empirical formula may be used. Trautwine gives, for preliminary estimates of the weights of steel trusses in pounds per square foot of building space covered, 0.05 to 0.09 × span in feet. The *roof covering* includes the jack rafters, purlins, sheathing, and the outer layer of felt and gravel, or slate, corrugated iron, etc. The weights, per square foot of roof surface, of these materials are given in works on this subject. The *snow-load* varies from 10 to 30 pounds per square foot of horizontal projection of the roof; it is about 20 pounds in the latitude of New York City; roofs inclined at 60 degrees or more have no snow-loads. The load due to wind pressure will be considered in Example 8.

Figure 11 shows one of the methods of *notation* used

for trusses. Within each triangle and between each pair of loads or forces, capital letters are placed, and the member or load is designated by the letters between which it lies. Thus, $BD$ is the lower section of the left rafter, $DE$ the adjacent diagonal, etc. The stresses in these members are indicated in the force polygon by the same letters, although not in capitals. Thus, the length of the side $bd$ gives the stress in the member $BD$, etc. The order in which the forces about a joint are taken in constructing the force polygon is frequently shown by a circle with arrowheads, as in the figure. At the left support $A$, and similarly at the right support, there are two opposing forces acting vertically: the half panel-load $W/2$ and the *total reaction* $R_1$ of the support, which reaction is equal to one-half the total load or $2\ W$. The difference, $1.5\ W$, between these two forces is the *effective reaction*.

Taking 13.5 pounds as the weight, per square foot of roof surface, of the roof covering and using the formula previously given for the truss weight, we have, for the truss shown in Fig. 11:

Area of roof surface = $12 \times 20.75 \times 2 =\ $ 498 square feet.

Weight of roof covering = $498 \times 13.5 = 6723$ pounds

Weight of truss = $0.07 \times 36\ (12 \times 36) = \underline{1244}$ pounds

Dead load                = 7967 pounds

Total reaction = $R_1 = R_2 = 7967/2 = 3984$ pounds

Panel-load = $W = 7967/4$      = 1992 pounds

On the load-line $aa'$, lay off, to any convenient scale of pounds to the inch, the loads in consecutive order; thus, $ab$ = load $AB = W/2$, $bc$ = load $BC = W$, etc. Joint 1 at the left support is in equilibrium under the

action of the load $AB$, the stresses in $BD$ and $DG$, and the reaction $GA$ or $R_1$. In the force polygon, $ab$ is the load $AB$ and $ga$ is $R_1$; from $b$ and $g$, lay off $bd$ and $gd$, parallel, respectively, to $BD$ and $DG$. The closed polygon for the joint is then $abdga$, and, in passing around it, $bd$ acts toward the joint and is compressive, while $dg$ acts from the joint and is tensile.

At joint 2, the system in equilibrium is composed of the stress $DB$ and the load $BC$ which are known, and the stresses in $CE$ and $ED$ which are to be determined. From $c$ and $d$, draw $ce$ and $ed$ parallel, respectively, to $CE$ and $ED$ and meeting at $e$. The closed polygon is then $bcedb$, and the character and magnitude of the stresses can be found as before. The similar polygon, similarly constructed, for joint 3 is $cc'e'fec$. Since both the truss and the system of loads are symmetrical, the stresses in corresponding members on the two sides are the same, and hence the stress diagram need not be considered further.

Tabulating the results, as measured to scale from the stress diagram, we have:

| MEMBER | STRESS DUE TO DEAD LOAD | |
|---|---|---|
| | Pounds | Kind |
| $BD$ . . . . . . . . | 6080 | − |
| $CE$ . . . . . . . . | 5080 | − |
| $ED$ . . . . . . . . | 1760 | − |
| $EF$ . . . . . . . . | 1760 | + |
| $FG$ . . . . . . . . | 3640 | + |
| $GD$ . . . . . . . . | 5280 | + |

The snow load is virtually an additional and temporary layer of roof covering. Hence, its weight produces the same kind of stress in each member as the dead load, the amount thus added to each dead load stress being proportional to the ratio of the snow and dead loads. Taking the weight of the snow as 15 pounds per square foot of projected area of the section of roof carried by the truss, the total snow-load is $36 \times 12 \times 15 = 6480$ pounds. The ratio, as above, is then $6480/7967 = 0.813$, and hence the total stress in any member, due to the two loads, is $1.813$ times that tabulated above.

8. ROOF TRUSS, WIND LOADS. The stresses produced in a roof truss by wind pressure are, in some cases, opposite in kind to those due to dead and snow loads. Therefore, a separate stress diagram is drawn for wind loads, and the results measured from it are added algebraically to the dead and snow load stresses, in order to obtain the final stress in each member. The latter is designed not only for the maximum stress of either kind, but also for withstanding stresses of both kinds, if the wind causes a reversal of stress.

The action of wind pressure on an inclined roof is not understood fully, and several empirical methods of estimating wind loads are used. Hutton's formula is:

$$p_n = p_h \sin \theta^{\,1.84 \cos \theta - 1}$$

in which $\theta$ is the inclination of the roof surface to the horizontal, $p_n$ is the perpendicular pressure per square foot on that surface and due to the wind, and $p_h$ is the

similar pressure on a vertical plane by a horizontal wind.*

The usual practice is to assume $p_h$ as 30 to 40 pounds. At 40 pounds and with $\theta = 30$ degrees, $p_n = 26\ 5$ pounds; and, remembering that only the windward side of the roof is subjected to wind pressure, we have, for the truss shown in Figs. 11 and 12:

Total load on windward side $= 20.75 \times 12 \times 26.5 = 6598.5$ pounds
Panel-load on windward side $= W_1 = 6598.5/2 \quad = 3299$ pounds

Again, metal trusses differ in their methods of support. If short, both ends may be fixed; if long and large, one end is fixed and the other is free to allow for expansion and contraction, the free end being supported on a roller or sliding bearing. If the truss is fixed at both ends, but one stress diagram for wind pressure need be made, with the wind on one side only, since, if the wind shift to the other side, the stresses in the two halves of the truss will simply be interchanged. When one end of the truss is free, however, the case is different, as, disregarding the friction of the bearing, the reaction at the free end is always vertical, while that at the fixed end varies in direction and magnitude as the wind shifts.

Let it be required to find the stresses due to wind

* A simpler expression of at least equal practical value is the Straight Line Formula:

$$P_n = AP/45$$

in which $A$, $P_n$, and $P$ correspond with $\theta$, $p_n$, and $p_h$ in Hutton's formula. The student will find additional expressions for the normal component of the wind pressure in works treating this branch of the subject in greater detail.

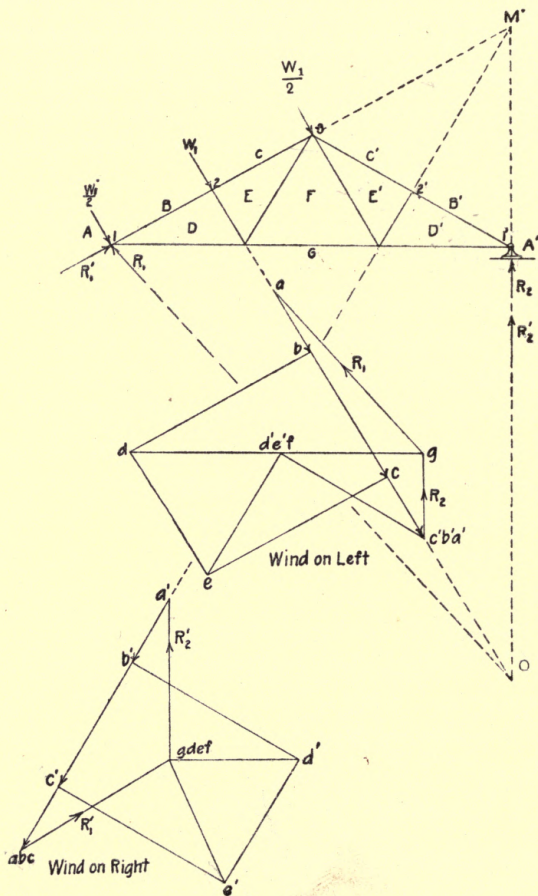loads on the truss whose dead load stresses were de-



FIG. 12

termined in Example 7. In Fig. 12, assuming the right end of the truss to be free and the wind to be on the

left, there will be a normal load, $W_1 = 3299$ pounds, at
joint 2, and a similar load of $W_1/2$ at each of the joints
1 and 3. As the dead and snow loads are not consid-
ered, there are no loads at joints $1'$ and $2'$. The
resultant of the wind loads must have the same line of
action as the middle load, since this load is the greatest
of the three and the two others are equal and equally
spaced. This resultant and the reactions at the two
supports form a system of external forces in equi-
librium, and hence the resultant of the two reactions
must be equal and opposite to that of the loads, and
have the same line of action.

As the right-hand end of the truss is free, $R_2$ will act
vertically. The lines of action of $R_2$ and the load-resultant
are thus known, as is the magnitude, $2W_1$, of the latter.
Prolong the line of action $2O$ of the resultant wind load
until it meets at $O$ that of $R_2$; then, since the forces are
concurrent, $O1$ is the line of action of $R_1$. On $2O$, lay
off the loads, $bc = W_1$ and $ab = cc' = W_1/2$. From $a$
and $c'$, draw $ag$ and $c'g$ parallel, respectively, to $O1$ and
$O1'$. The force polygon for the loads and reactions is
then $ac'ga$ and $ac' =$ loads, $c'g = R_2$, and $ga = R_1$.

Joint 1 is in equilibrium under the action of $R_1$, the
load $W_1/2$, and the stresses in the members $BD$ and
$DG$. In the force polygon, $ab$ is the load; draw $bd$
and $gd$ parallel to $BD$ and $GD$, respectively. The force
polygon is then $abdga$, and $bd$ is a compressive, and $dg$
a tensile, stress.

At joint $1'$, the system in equilibrium is the reaction
$R_2$ and the stresses in $B'D'$ and $D'G$. As there are no

loads at joints $1'$ and $2'$, the vertex $c'$ in the force poly-
gon is also the location of the points $b'$ and $a'$. Draw
$b'd'$ and $gd'$ parallel to the two members. The force
polygon is then $a'gd'b'a'$, and $gd'$ is tensile and $d'b'$
compressive.

Proceeding similarly for joint 2, the force polygon is
$bcedb$. In the polygon for joint $2'$, $b'd'$ is known and $b'$
and $c'$ lie at the same point. Hence, $e'c'$ must coincide
with $b'd'$, and the force polygon is $b'd'e'c'b'$, which is a
straight line. There is no stress in the diagonal $D'E'$.

In the polygon at joint 3, there are known the three
consecutive sides, $ec$, $cc'$, and $c'e'$; the remaining sides,
$e'f$ and $fe$, must meet at the point $f$. Hence, $f$ coincides
with $e'$, and there is no stress in the member $E'F$. The
polygon is $cc'e'fec$.

Now, let the wind shift to the right. The loads (not
shown) due to it will be removed to joints $1'$ and $2'$, and
that at joint 3 will be parallel to them and normal to the
right rafter. Their resultant will pass through $2'$ and
meet the new right reaction $R'_2$ — which is vertical as
before — at the point $M'$. Hence, $M'1$ is the line of
action of the new left reaction $R'_1$, which line practi-
cally coincides with the rafter, as the inclination of the
latter is about 30 degrees. Prolonging the line $M'2'$
for the load-line $a'c$ and proceeding as before, the force
polygon $a'cga'$ for the loads and reactions is drawn.
The stress diagram is laid out by the previous method,
except that the stresses are taken in non-clockwise order.

Tabulating the results, as measured from the dia-
grams in Figs. 11 and 12, we have:

## FINAL STRESSES

| Member | Dead Load | Snow Load | Wind on | | Stress | |
|---|---|---|---|---|---|---|
| | | | Fixed Side | Free Side | Maximum | Minimum |
| $BD$ | − 6080 | − 4943 | − 4600 | − 3760 | − 7.86 | − 3.04 |
| $B'D'$ | − 6080 | − 4943 | − 3800 | − 4440 | − 7.73 | − 3.04 |
| $CE$ | − 5080 | − 4130 | − 4600 | − 3760 | − 6.9 | − 2.54 |
| $C'E'$ | − 5080 | − 4130 | − 3800 | − 4440 | − 6.82 | − 2.54 |
| $DE$ | − 1760 | − 1431 | − 3299 | 0 | − 3.24 | − 0.88 |
| $D'E'$ | − 1760 | − 1431 | 0 | − 3299 | − 3.24 | − 0.88 |
| $EF$ | + 1760 | + 1431 | + 3200 | 0 | + 3.19 | + 0.88 |
| $E'F$ | + 1760 | + 1431 | 0 | + 3120 | + 3.16 | + 0.88 |
| $DG$ | + 5280 | + 4293 | + 6640 | 0 | + 8.1 | + 2.64 |
| $D'G$ | + 5280 | + 4293 | + 3260 | + 2920 | + 6.41 | + 2.64 |
| $FG$ | + 3640 | + 2959 | + 3260 | 0 | + 4.93 | + 1.82 |

The maximum and minimum stresses are given in tons (2000 pounds), the remainder in pounds. The maximum stress is, in each case, the algebraic sum of the dead and snow loads and the greater of the two wind loads ; the minimum is the dead load stress. As before, tensile stresses are marked + and compressive stresses −.

9. CRANE TRUSS. Trusses are used frequently in crane construction. Thus, the crane shown in Fig. 13 is virtually a braced cantilever, the vertical and diagonal members serving to stiffen the structure and to distribute the dead load, which is the weight of the crane. The live load $W_1$, suspended from the peak $A'$, is, from the peak to the diagonal $PQ$, assumed to be taken solely by the upper and lower chords or sides of the boom, the
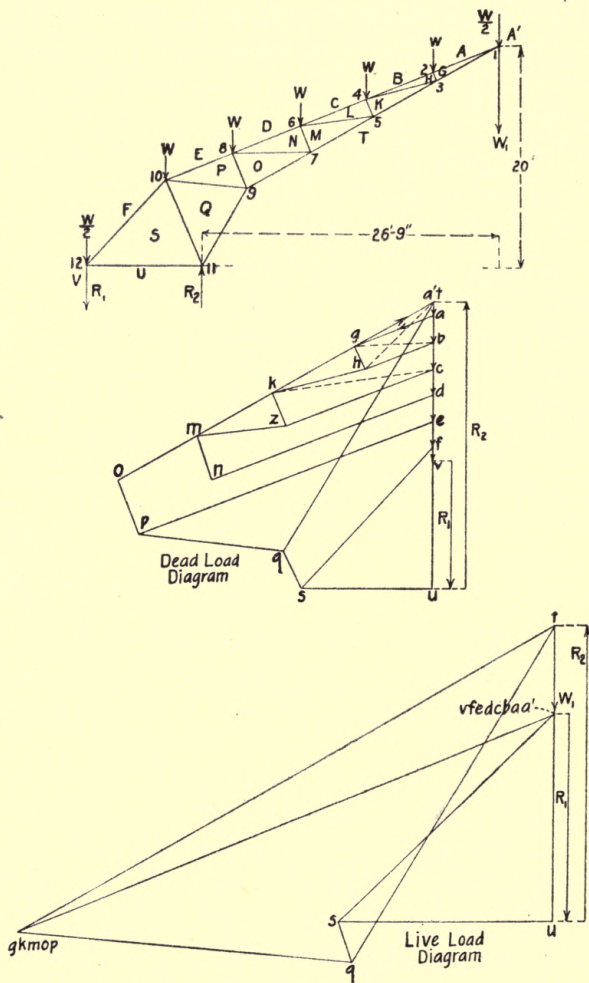
FIG. 13

section between joints 1 and 10 acting as a tie and that from 1 to 9 as a strut. This assumption is sufficiently correct for general purposes, but, for an absolutely accurate determination of the stresses in the members, the effect of bending should be considered also.

Let the total weight of the crane or dead load be 1800 pounds and the maximum live load or weight to be carried at the peak be $W_1 = 5$ tons (10,000 pounds). Assume the dead load to be distributed among the upper joints, as in a roof truss, the panel-load $W$ being thus 300 pounds. While this distribution is not strictly that which actually exists, it is generally used, for cranes of moderate size, as a ready and reasonable approximation.

A single stress diagram could be drawn for both the dead and live loads, if the load at joint 1 were taken as the total load there, or $W_1 + W/2$; but $W_1$ is so much greater than $W$ that the scale necessary for accurate measurement of the stresses in the members would make the diagram inconveniently large. Hence, separate diagrams, at different scales, will be drawn for the dead and live loads. The total stress in a member will then be the algebraic sum of those given by the two diagrams.

In the dead load diagram, lay off the loads from $a'$ to $v$, taking them in anti-clockwise order. As $W_1$ is disregarded, the point $a'$ corresponds also with the point $t$. Joint 1 is in equilibrium under the action of the load $A'A$ and the stresses in the members $AG$ and $GT = GA'$. Drawing the stress lines parallel

to their corresponding members, we have the force triangle $aga'$, in which $ag$ is a tensile, and $ga' = gt$ is a compressive stress.

At joint 2, the known forces are $GA$ and $AB = W$; their resultant is $gb$. From $g$ and $b$, lay off $gh$ and $bh$ parallel, respectively, to $HG$ and $BH$ and meeting at $h$. The force polygon is then $gabhg$. The brace $HG$ thus acts as a strut, which is the case with the similar braces parallel to it.

The known stresses at joint 3 are $a'g$ and $gh$; their resultant is $a'h$. From $a'$ and $h$, lay off $a'k$ and $hk$ meeting at $k$. The force polygon is then $a'ghka'$, and $HK$ is a tie, as are $LM$ and $NO$. The polygons for joints 4 to 8 are drawn similarly.

At joint 9, the known stresses are $a'c$ and $op$; their resultant is $a'p$. Lay off $a'q$ and $pq$ parallel to their corresponding members. The force polygon is $a'opqa'$, and $PQ$ is a strut.

The force polygon for joint 10 is $qpefsq$, and $SQ$ is a strut. At joint 11, the polygon is $a'qsua'$; $SU$ is a strut, and $ua' = R_2$, the right and upward reaction from the foundation. The similar polygon for joint 12 is $usfvu$, and $vu = R_1$ is the downward pull of the foundation bolts.

Now, disregard the weight of the crane and consider only the effects of the live load $W_1$ at the peak. The general method and the order in which the stresses are taken are the same as before. In the live load diagram lay off $W_1$ from $t$ to $a'$. Joint 1 is in equilibrium under the action of $W_1$ and the stresses in the

members $AG$ and $GT$. As there are no dead loads at
the intervening joints, each of these stresses is uniform
in intensity throughout the members from joint 1 to
joints 10 and 9, respectively, and therefore the point $a'$
coincides with the points $a$, $b$, $c$, $d$, $e$, and $f$, and simi-
larly the point $g$ corresponds with the points $k$, $m$, $o$,
and $p$. From $t$ and $a$, lay off $tg$ and $ag$, parallel to $GT$
and $AG$, respectively, and meeting at $g$. The force
polygon is then $tagt$; $ag$ is the tensile stress in the
upper members between joints 1 and 10; $gt$ is the stress
in the members forming a strut between joints 1 and 9;
and there is no stress, except that due to bending, in the
diagonal braces from joint 1 to, and including, the
member $OP$.

At joint 9, the stresses acting are those in the mem-
bers $TO$, $PQ$, and $QT$, that in $OP$ being zero. The
force polygon is $topqt$, and $PQ$ is a strut. The similar
polygon for joint 10 is $qpefsq$, in which the stress $pe=ga$
and $ef$ is zero. $FS$ and $SQ$ are under tension. For
joint 11, the polygon is $tqsut$; $SU$ acts as a strut and $ut$
gives the magnitude of $R_2$, the right reaction and up-
ward pressure from the foundations. Finally, the poly-
gon for joint 12 is $usfvu$, in which the point $v$ coincides
with $f$, and $vu = R_1$ is the downward pull of the founda-
tion bolts.

Tabulating the stresses, as measured from the two
diagrams, and taking the algebraic sum to obtain the
total stress in each member, we have:

| Member | Stress (Lb.) due to | | Total Stress | Member | Stress (Lb) due to | | Total Stress |
|---|---|---|---|---|---|---|---|
| | Dead Load | Live Load | | | Dead Load | Live Load | |
| AG | + 940 | + 65,000 | + 65,940 | GH | — 280 | 0 | — 280 |
| BH | + 820 | + 65,000 | + 65,820 | KL | — 400 | 0 | — 400 |
| CL | + 1780 | + 65,000 | + 66,780 | MN | — 530 | 0 | — 530 |
| DN | + 2640 | + 65,000 | + 67,640 | OP | — 660 | 0 | — 660 |
| EP | + 3540 | + 65,000 | + 68,540 | QS | — 480 | + 3500 | + 3020 |
| FS | + 2170 | + 34,000 | + 36,170 | HK | + 940 | 0 | + 940 |
| GT | — 1000 | — 69,500 | — 70,500 | LM | + 1010 | 0 | + 1010 |
| KT | — 2080 | — 69,500 | — 71,580 | NO | + 1100 | 0 | + 1100 |
| MT | — 3060 | — 69,500 | — 72,560 | PQ | — 1640 | — 37,000 | — 38,640 |
| OT | — 4080 | — 69,500 | — 73,580 | SU | — 1480 | — 23,500 | — 24,980 |
| QT | — 3240 | — 44,500 | — 47,740 | | | | |

Similarly, the reactions are :

| | Dead Load | Live Load | Total, Pounds |
|---|---|---|---|
| $R_1$ | 1450 | 24,000 | 25,450 |
| $R_2$ | 3250 | 34,000 | 37,250 |

# CHAPTER III

**7. Beams.** In the analysis of their stresses, many machine and structural members are classed as 'beams,' since the effects of their applied loads are the same in character as those produced by the loads on a beam. Technically, a *beam* is a rigid bar, generally horizontal, and sustained by one or more supports. A *simple beam* rests freely on two supports, one at each end; a *continuous beam* has three or more supports; a *cantilever beam* has but one support, which is at one end, this definition including the part of any other form of beam which projects beyond the support which upholds it.

Supports may be either *free* or *restrained*. With the former, the beam simply lies on its supports; when the beam is restrained, its end or ends are built in or otherwise so fixed that the tangent to the elastic curve at the supports is horizontal. The *elastic curve* is the curve into which the neutral axis or neutral surface of the beam is bent by the loads. This bending produces compression on one side of the beam and tension on the other; the *neutral axis* lies in the plane of zero stress between the compressed and stretched parts and passes through the centre of gravity of the cross-section of the beam.

**8. Vertical Shear.** The principal effects of the loads on a beam are to produce vertical shear and bending moments. Thus, Fig. 14 (*A*) represents a simple beam
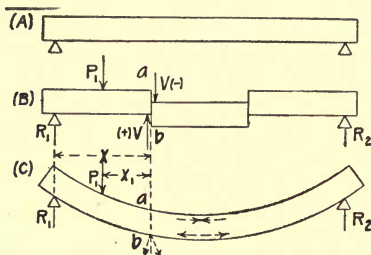


FIG. 14

without loads. When under load, it is subjected simultaneously to shearing, as in (*B*), and to bending, as in (*C*).

At any section, as *ab*, it will be seen that, in shearing, the segment of the beam to the left of *ab* is forced vertically upward and that to the right vertically downward by two equal and opposite forces acting at the section, each known as the *vertical shear V.* The shear to the left is that usually considered; it is equal to the resultant or algebraic sum of the forces to the left of any section as *ab*, upward forces and shears being considered as positive, and those acting downward negative.

Thus: $$V = R_1 - \Sigma P,$$

in which $R_1$ is the left reaction and $\Sigma P$ is the algebraic sum of the upward and downward forces between the section and the left support. In (*B*), if the weight of the segment of the beam to the left is neglected, $\Sigma P = P_1$; if this weight is considered, $\Sigma P = P_1 + wx$, in which $w$ is the weight of the beam per unit of length and $x$ is the distance of the section *ab* from the left support.

**9. Bending Moment.** Similarly, at any section, as *ab* in ($C$), Fig. 14, the forces to the left of the section combine to rotate the left segment in a clockwise direction, while those to the right act, with an equal resultant *bending moment*, to produce rotation in the reverse direction. The results are to compress the upper side of the beam, stretch the lower side, and, if the load be excessive, to rupture the fibres at the bottom, and ultimately the beam. These actions are the effects of the moments, about any point in the section *ab*, of the forces acting to the left or right of that section, the resultant bending moment of the forces to the right being equal to that of those on the left.

Since the bending moment is the product of a force by its distance from the section, it is a compound quantity, expressed in pounds-inches or tons-feet. Neglecting the weight of the beam, the bending moment at the section *ab* is the algebraic sum of the moments of the forces to the left of that section, or :

$$M = R_1 \times x - P_1 \times x_1.$$

The weight of the segment to the left will act at the centre of gravity of that segment. Considering this weight, the bending moment becomes :

$$M = R_1 \times x - P_1 \times x_1 - wx \times x/2.$$

**10. Resisting Moment.** The *resisting moment* at any section of a beam is the algebraic sum of the moments of the internal horizontal stresses about any point in the section considered, these stresses being produced by the

bending moment. For equilibrium, the bending moment and resisting moment must be equal. Hence, as is shown in works on Strength of Materials:

Resisting Moment $= SI/c =$ Bending Moment $= M$, in which $S$ is the unit-stress on the fibre most remote from the neutral axis of the cross-section of the beam at the section considered, $c$ is the distance of that fibre from the neutral axis, and $I$ is the rectangular moment of inertia of the cross-section.

**11. Shear and Moment Diagrams.** The vertical shear and bending moment at any section of a beam can be found from the force and equilibrium polygons. Thus,



FIG. 15

let $AB$, Fig. 15, represent a simple beam, having three vertical loads of 100, 400, and 250 pounds, spaced as shown. Lay off these loads on the load-line $ad$, draw

the rays $Oa$, $Ob$, $Oc$, and $Od$, and construct the corresponding equilibrium polygon $CDEFGC$, the closing side $GC$ of which determines the magnitude $ea$ of the left reaction $R_1$ and the similar magnitude $de$ of $R_2$.

The vertical shear to the left of any section of a beam is, by definition, equal to the left reaction, minus the sum of the loads to the left of that section. Hence, between the left support and the load $P_1$, $V = R_1 = ea$; between $P_1$ and $P_2$, $V = R_1 - P_1 = ea - ab = eb$; between $P_2$ and $P_3$, $V = R_1 - (P_1 + P_2) = ea - ac = -ec$; and, between $P_3$ and the right support, $V = R_1 - (P_1 + P_2 + P_3) = ea - ad = -ed = -R_2$. Projecting these values from the load-line on the lines of action of the forces, we have the vertical shear diagram $efkgh$. The zero-line of this diagram is $eg$, the shear above it being positive or upward, and that below it, negative and downward.

Now, let it be required to find the bending moment at any section, as $T$, of the beam, the corresponding ordinate of the equilibrium polygon being $tt'$. The magnitude of the bending moment depends on the magnitudes and locations of the forces to the left of $T$; these forces are $P_1$ and $R_1$, whose resultant $r$ is equal to $eb$ and acts vertically upward through a point yet to be determined.

Assume the polygon to be cut on the line $tt'$. Then, to maintain equilibrium, the stresses $S$ and $S'$ in the sides $Ct'$ and $Dt$, respectively, must have a resultant which is equal and opposed to $r$ and which has the same line of action. The resultant $r$ and these two stresses thus form a system of concurrent forces in equilibrium,

and hence their lines of action must meet at the common point $m$, the intersection of $t'C$ and $tD$, prolonged. The line of action of $r$ therefore passes through the point $m$, distant $l$ from $T$, and the bending moment $M$ at the section $T$ is $r \times l$.

Let $H$ be the pole-distance in the force polygon. The triangles $mtt'$ and $Oeb$ are similar. Hence :

$$eb : tt' :: Oe : mt$$
$$r : tt' :: H : l$$
$$r \times l = tt' \times H = M.$$

That is, as $H$ is the same for all the forces and as the ordinate $tt'$ is parallel to the lines of action of the latter, *the bending moment at any section of the beam is proportional to the corresponding ordinate of the equilibrium polygon, and is equal to the product of the length of that ordinate by the pole-distance.* This principle holds only when the lines of action of the forces acting on the beam are parallel and the section considered is parallel to those lines.

Under these conditions, the bending moment diagram is the equilibrium polygon for the forces acting on the beam. Figure 15 represents this diagram for concentrated loads. If, as in Fig. 16, the beam is covered by a uniform load of $w$ pounds per unit of length and the span be $s$, the left reaction will be one-half of the total load, or $ws/2$, and, at any section, distant $x$ from the left support, the bending moment will be :

$$M = wsx/2 - wx \times x/2$$

which is the equation of a parabola whose maximum ordinate $ws^2/8$ occurs at the middle of the beam, where $x = s/2$.

As with the vertical shear, the bending moment may be either positive or negative. For positive moments, the diagram is usually drawn above the zero-line, and, for negative moments, below it, although this practice is not invariable.

**12  Moment Scale.** To make practical use of the equilibrium polygon as a bending moment diagram, the scales of the force and equilibrium polygons must be considered. Thus if, in the equilibrium polygon, Fig. 15, the spacing of the forces be on a *linear scale* of 4 feet to the inch, or 48 to 1, the length of the ordinate $tt'$, as measured in inches from the diagram, must be multiplied by 48 to give the result in pound-inches, or by 4 to obtain it in pound-feet. Again, if the force polygon be laid out on a *force scale* of 400 pounds to the inch, and the measured length of the pole distance $H$ be $1\frac{1}{4}$ inches, then $H = 1.25 \times 400 = 500$ pounds. The *moment scale*, by which $tt'$ and similar ordinates are to be measured, is then the *product of the linear scale by the pole distance measured on the force-scale* or, in this case:

$$4 \times 500 = 2000 \text{ pound-feet per inch,}$$

to obtain $M$ in pound-feet, or

$$48 \times 500 = 24{,}000 \text{ pound-inches,}$$

to obtain $M$ in pound-inches.

On the linear scale, as above, the measured length of
$tt'$ in the original drawing was 0.725 inch, and the
bending moment $M$ at the section $T$ is therefore

$$0.725 \times 2000 = 1450 \text{ pound-feet,}$$
or $\qquad 0.725 \times 24{,}000 = 17{,}400 \text{ pound-inches.}$

**13. Twisting Moments.** Shafting transmits power by
torsion. The driving pulley pulls forward, the driven
pulley drags backward, and hence the shaft between
them is twisted by an amount depending on its material
and the resistance to its forward motion. For equilib-
rium, the moment of the driving force must be equal
to that of the resistance, and therefore the twisting
moment is uniform throughout the shaft between the
driving and driven pulleys.

The effect of torsion is to produce shearing stress on
the shaft in planes perpendicular to the axis. The
moment of resistance of the shaft to twisting is $S_s J/c$,
which is evidently equal to the driving moment. $S_s$ is
here the unit shearing stress at the distance $c$ of the
most remote fibre from the axis, *i.e.* at the outer extrem-
ity of the radius, and $J$ is the polar moment of inertia
of the cross-section of the shaft. For a solid cylin-
drical shaft of diameter $d_1, J = \pi d_1^4/32$; for a hollow
cylindrical shaft of external diameter $d_1$ and internal
diameter $d, J = \pi(d_1^4 - d^4)/32$.

The driving moment — and hence its equal, the twist-
ing moment — may be expressed, as with bending mo-
ments, in pound-inches, pounds- or ton-feet, etc. Since
the twisting moment is uniform throughout the length

of shafting under torsion, its diagram is simply a rect-
angle of that length and having a height equal to the
moment expressed in pound-inches, etc., to any given
moment-scale, as shown in Fig. 19.

**14. Bending and Twisting Moments combined : Equiva-
lent Bending and Twisting Moments.** Shafts may be
subjected to both twisting by the driving moment and
to bending by the weights of gears and pulleys, the
pull of belt-tensions, etc.   In such cases, in place of
finding the stress due to each of these actions separately
and taking their sum, it is more convenient to compute
the stress corresponding with an assumed bending or
twisting moment which is equivalent in its effect to
the original bending moment and the twisting moment
combined.

Let $M_b$ be the bending moment, $M_t$ the twisting mo-
ment, and $EM_b$ the equivalent bending moment.   Then,
it can be shown, by the relation between the polar and
rectangular moments of inertia for a circular cross-sec-
tion, that :

$$EM_b = \tfrac{1}{2}(M_b + \sqrt{M_b^2 + M_t^2}).$$

Similarly, letting $EM_t$ be the equivalent twisting mo-
ment, we have :

$$EM_t = M_b + \sqrt{M_b^2 + M_t^2}.$$

These expressions hold only for solid cylindrical bodies,
like a shaft.

Bending moments, when laid out by the same moment
scale, can be treated exactly like forces, *i.e.*, a line can

E

be drawn representing each moment, and a force poly-
gon constructed for a series of these moments. The
resultant bending moment of two or more such mo-
ments in different planes can thus be found. Simi-
larly, twisting and bending moments, drawn to the same
moment scale, can be combined to form the equivalent
bending or twisting moment by applying the equations
just given. Example 13 shows the methods for these
combinations.

### EXAMPLES

10. *I*-Beam: Uniformly Distributed Load. — The
methods of treating a uniformly distributed load are an
extension of those used for concentrated loads, as shown
in Fig. 15. When the load is uniform over the entire
span of a beam, the number of equal loads or parallel
forces becomes indefinitely large, since the weight of
each elementary particle of the load acts at the centre
of gravity of that particle. In finding the bending
moment at any section of the beam, these elementary
forces to the left of that section may be replaced by
their resultant, which acts at the centre of gravity of
the total load to the left and is equal in magnitude to
that load.

Thus, in Fig. 16, let $AB$ be a simple beam of span $s$
and carrying a uniformly distributed load of $w$ pounds
per unit of length. The total load is then $w \times s = W$
the left reaction is $W/2 = ws/2$, and the bending mo-
ment at any section $a$, distant $x$ from the left support,
is (Art. 9):

$$M = ws/2 \times x - wx \times x/2,$$

since $wx$ is the total weight of that portion of the load between $a$ and $A$, $x/2$ is the distance of the centre of gravity of $wx$ from $a$, and hence the lever-arm of that weight, and $x$ is the similar arm of the left reaction $R_1$. This expression for $M$ is the equation of a parabola, as $A'a'B'$, whose ordinate is zero at each support and a maximum at mid-span. At the latter point, $x = s/2$ and $M = ws^2/8 = Ws/8$.



FIG. 16

The curve of bending moments can be drawn by computing the value of the moment at mid-span, laying off this value there to any suitable moment scale, and then describing a parabola through the point $D'$ thus obtained, and the points $A'$ and $B'$. If the total load be divided into any number, as three equal parts, vertical lines of action be drawn through the centres of gravity of these parts, the three equal loads be set off on the load-line $bc$, and the force and equilibrium polygons, $Obc$ and $A'C D'E'B'$, be constructed, it will be found that the points $C'$, $D'$, and $E'$ will lie on the parabolic curve. If the load be divided into an indefinitely large number of equal parts, the equilibrium polygon and the curve will coincide.
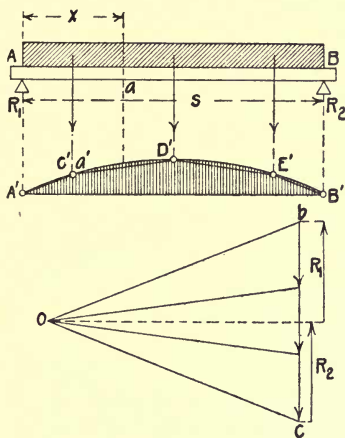
As an example, take a 15-inch $I$-beam of 16 feet span,

weighing 50 pounds per foot of length, and carrying a uniformly distributed load of 43,000 pounds, including its own weight. It is required to determine the maximum fibre stress in pounds per square inch.

The maximum stress will occur at mid-span, where $M = Ws/8$. By the common theory of flexure, $M$ is also equal to $SI/c$ (Art. 10). Equating:

$$SI/c = Ws/8 = M,$$

in which $S$ is the maximum stress, $I$ is the rectangular moment of inertia of the cross-section of the beam about a neutral axis passing through the centre of gravity of that cross section and perpendicular to the web, $c = 15/2 = 7.5$ is the distance of the most remote fibre from the neutral axis, $W = 43,000$ pounds, and $s = 16$ feet $= 192$ inches. For this beam, $I = 483.4$, and hence $I/c = 64.5$. Substituting:

$$S \times 64.5 = 43,000 \times 192/8$$
$$S = 16,000 \text{ pounds per square inch.}$$

The maximum bending moment is

$$S \times 64.5 = 1,032,000 \text{ pound-inches,}$$

which is the moment given by the ordinate at $D'$, Fig. 16, when that ordinate is measured by the moment scale computed as described in Art. 12.

11. LOCOMOTIVE SIDE ROD WITH UNIFORM LOAD DUE TO CENTRIFUGAL FORCE. The parallel or side rod connecting the two driving wheels of a locomotive, is an example of a simple beam carrying a uniformly distributed load produced by centrifugal force. Every

point in the rod is revolving in a circle whose radius is the length of the crank. When a body of weight $W$ rotates about an axis in a circle of radius $r$ with a uniform velocity $v$, the centrifugal force acting radially outward on the body is:

$$F = Wv^2/gr,$$

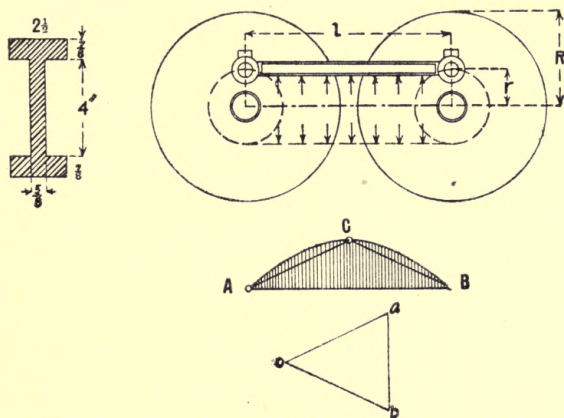in which $g$ is the acceleration of gravity, or 32.2 feet per second per second.



FIG. 17

On each elementary particle of the revolving side rod, there is acting, therefore, a centrifugal force parallel to the crank. These forces have their maximum effect in bending the rod, when the latter is in its highest or lowest position, as shown in Fig. 17, and the lines of action of the forces are perpendicular to the axis of the rod; when the rod is on the line of centres, the forces

act along its axis to produce direct tension or compression; at any intermediate position, their effect on the rod is partly flexural and partly to produce direct stress, as above.

Let $V$ be the circumferential velocity of the driving wheels, which is that of the train; $v$, the corresponding velocity of the crank pins; and $R$ and $r$, the radii of the driving wheels and the crank pin circles, respectively. Then:

$$v : V :: r : R ;$$
$$v = \frac{Vr}{R}.$$

Again, let $w$ be the weight of a unit of length of a side rod of length $l$, and let $f$ be the corresponding centrifugal load when the rod is in its highest or lowest position and the bending effect is a maximum. Then (Example 10), the maximum bending moment is

$$M = fl^2/8 = Fl/8 = SI/c.$$

But

$$f = wv^2/gr = wrV^2/gR^2.$$

Substituting,

$$M = wrV^2l^2/8 \, gR^2,$$

and

$$S = Mc/I = (wrV^2l^2/8 \, gR^2) \times c/I,$$

which are the expressions for the bending moment and maximum stress at the mid-length of the rod.

In the force polygon, Fig. 17, let $ab = F = fl =$ the total centrifugal load. Construct the force polygon $Oab$ and the corresponding equilibrium polygon $ACB$, and, through the points $A$, $C$, and $B$ draw a parabolic

curve. The bending moment diagram, for the highest and lowest positions of the rod, will then be the area included between the curve and the zero line $AB$.

As an example, take a locomotive with cylinders of 20 inches diameter and 26 inches stroke; driving wheels, 6 feet 8 inches diameter; side rods, 7 feet 2 inches long between centres and of I-section, $2\frac{1}{2} \times 5\frac{3}{4}$ inches, as shown in Fig. 17; train speed, 80 miles per hour.

In substituting in the expressions for $M$ and $S$, the values must be given in the same units of length and time. Taking the inch and the second as these units, $g = 32.2 \times 12 = 386.4$ inches per second per second; $V = 80$ miles per hour = 117.3 feet per second = 1407.6 inches per second; $R = 80/2 = 40$ inches; $r = 26/2 = 13$ inches; $v = rV/R = 13 \times 1407.6/40 = 457.47$ inches per second; $l = 86$ inches; and the weight of the steel rod (except ends) per inch of length is $w = 1.93$ pounds. The moment of inertia $I$ of the cross-section of the rod about an axis perpendicular to the web is 29.606; $c = 2.875$ inches; and the section modulus $I/c = 10.29$ and its reciprocal $c/I = 0.097$.

Substituting in the formulas, we have: centrifugal force per inch of length of bar $= f = 80.48$ pounds; total centrifugal load $= F = 80.48 \times 86 = 6921$ pounds; bending moment at mid-length of bar (ordinate at $C$, Fig. 17) $= M = 6921 \times \frac{86}{8} = 74,401$ pound-inches; and the maximum stress at mid-length $= S = 74,401 \times 0.097 = 7217$ pounds per square inch, a result which is low owing to the fact that the rod is short and its moment of inertia is relatively large. With some rods, 100 inches

or more in length, this stress at this speed is approximately 14,000 pounds per square inch.

It will be understood that these calculations refer only to the stress produced by centrifugal force. There is, in addition, a direct compression or tension in the rod,
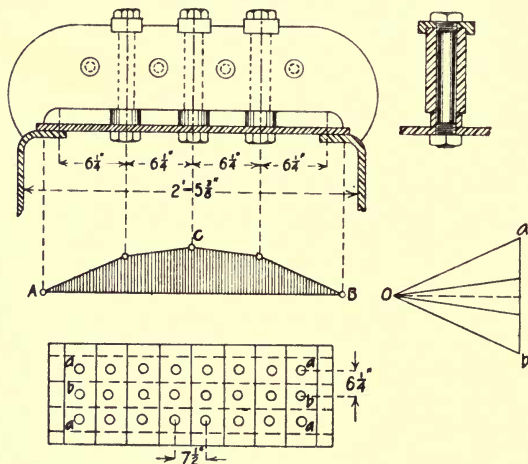


FIG. 18

whose maximum value, with two driving wheels on each side, is one-half of the greatest thrust on the piston. The rod, when under compression, acts as a column, and its cross-sectional area should be sufficiently great to keep the direct compressive stress below 5000 pounds per square inch. Finally, there may be various indeterminate stresses, due to lack of alignment, and momentarily, to unequal adhesion of the driving wheels.

12. GIRDER STAY WITH STRESSES PRODUCED BY ITS

SUPPORTING A CONTINUOUS BEAM UNDER UNIFORM LOAD. Figure 18 represents a girder stay for the top of the combustion chamber of a marine cylindrical boiler. The stay is a beam of 25 inches span and is formed of two steel plates, each $\frac{7}{8} \times 7\frac{1}{2}$ inches; the stays are spaced $7\frac{1}{2}$ inches apart, centre to centre. The working pressure is 160 pounds, gauge. The resultant load on the top of the combustion chamber is transmitted to each stay by three bolts, spaced $6\frac{1}{4}$ inches apart, and $1\frac{17}{64}$ inches diameter at the root of the thread. Each bolt passes through a yoke-plate at the top which divides the load between the two parts of the stay and also keeps them from separating. It is required to find the maximum stress in the two plates of the stay.

An approximate method which is sometimes used, is to assume that the stay, as a simple beam, carries, through each of its three bolts, one-fourth of the total load on the strip of 25 inches span and $7\frac{1}{2}$ inches width, the remaining fourth being divided equally between the tube-sheet and the back plate of the combustion chamber. On this basis, we have, for each stay:

Total load on braced strip = $25 \times 7.5 \times 160 = 30{,}000$ lb.
Load on each bolt = span load between bolts = $30{,}000/4 = 7{,}500$ lb.
Load on tube sheet = load on back plate = $7500/2 = 3{,}750$ lb.

The problem then reduces to that of a simple beam $AB$, Fig. 18, having three concentrated and approximately equally spaced loads, each of 7500 pounds, and hence an effective reaction at each support of $7500 \times \frac{3}{2}$ = 11,250 pounds. Constructing the force and equilib-

rium polygons, $Oab$ and $ACB$, the maximum bending moment at $C$ is found, by measuring the ordinate there, to be 93,750 pound-inches $= M = SI/c$ (Art. 10).

For a rectangle of base $b$ and depth $d$, $I = bd^3/12$ and $c = d/2$. Hence $I/c = bd^2/6$ and $S = 6M/bd^2$. In this case, the two plates of the girder are virtually combined, since the yoke-plate and the central line of rivets prevent buckling. Therefore, $b = \frac{7}{8} \times 2 = 1.75$ inches, $d = 7.5$ inches, and

$$S = \frac{6 \times 93,750}{1.75 \times 7.5 \times 7.5} = 5714 \text{ pounds per square inch,}$$

which is the maximum stress in the girder, since the ordinate at $C$ gives the maximum bending moment.

This method is, as stated, approximate. For an accurate determination of the stresses, the top of the combustion chamber should be treated as a series of continuous beams, the end supports of which are the walls of the chamber, the bolts being the intermediate supports. The reactions at these intermediate supports form the loads on the bolts, and, when these loads are found, the girder may be treated, as before, as a simple beam.

Referring to the top plan of the combustion chamber in Fig. 18, it will be seen that there are 8 equally spaced stays, each supporting a transverse section, $7.5 \times 25$ inches, of the top, which section is virtually a continuous beam, supported from above. In this section, there are 4 equal spans, $6\frac{1}{4}$ inches long, the load on each being $6.25 \times 7.5 \times 160 = 7500$ pounds. In the transverse direction, parallel to the stay, the top of the chamber

will sag under the load and between the bolts, thus producing bending moments in that direction.    Let $w =$ span load $= 7500$ pounds.    Then, according to the *Theorem of Three Moments*, the reactions at the 5 supports are :

| Support | 1 | 2 | 3 | 4 | 5 |
|---------|---|---|---|---|---|
| Reaction | $11\,w/28$ | $32\,w/28$ | $26\,w/28$ | $32\,w/28$ | $11\,w/28$ |

It will be seen that the distribution of the load is not uniform and that the greater loads come on the second and fourth supports, *i.e.*, on the two outer bolts, *aa*, of the stay.    Each of these bolts supports one-half of each of the adjacent spans of the transverse strip, so that, owing to the bending stress, the area thus supported may be considered as having a load of $w_1 = 32\,w/28$ $= 8571$ pounds.    The load on the similar central area and bolt is $w_2 = 26\,w/28 = 6964$ pounds, and the total load on the 5 supports is $112\,w/28 = 30,000$ pounds, as previously computed.

Again, the top of the combustion chamber will sag, under the load and between the bolts, in the longitudinal direction, at right angles to the stays.    It may, therefore, be considered as divided in that direction into 3 continuous beams, each with 9 equal spans and 10 supports.    The reaction at support 10 is the same as that at 2, etc.    Neglecting the load just computed for transverse bending, we have, for the original span load $w$ (*Theorem of Three Moments*):

| Support | 1 | 2 | 3 | 4 | 5 |
|---------|---|---|---|---|---|
| Reaction, | $209\,w/530$, | $601\,w/530$, | $511\,w/530$, | $535\,w/530$, | $529\,w/530$ |

The greater loads are therefore those on supports 2 and 9, which supports, for the 3 beams, are the 3 bolts in each of the two end girders of the series. These two stays are therefore those which are under the maximum stress.

Considering both transverse and longitudinal bending, the resultant loads on the bolts of the two end girders are:

On each outer bolt, $a$, load $= (w \times 32/28)(601/530) = 1.29\,w = 9675$ lb.

On middle bolt, $b$, load $= (w \times 26/28)(601/530) = 1.05\,w = 7875$ lb.

That is, the outer two of the three continuous beams assumed for longitudinal bending stresses are considered as having a span load throughout of $w_1 = 8571$ pounds, and similarly, the middle beam, a uniform span load of $w_2 = 6964$ pounds.

Treating the girder as a simple beam and following the approximate method in all respects, except the changes as above in the bolt loads, the maximum stress $S$ in the girder is found to be 6686 pounds per square inch, or 17 per cent more than that given by the approximate method. These calculations apply only to the two girder stays at the ends of the row; the intermediate girders are under less stress, as is shown by the coefficients for the reactions at the corresponding supports. The values given for these coefficients refer only to beams of constant cross-section, under a uniform load, and with all supports at the same level. Strictly speaking, they are limited also to beams with free ends (Art. 7), although the difference in this respect is not material in this case.

13. COUNTER SHAFT: TWISTING AND BENDING COMBINED. Figure 19 represents a counter shaft $AB$, sup-
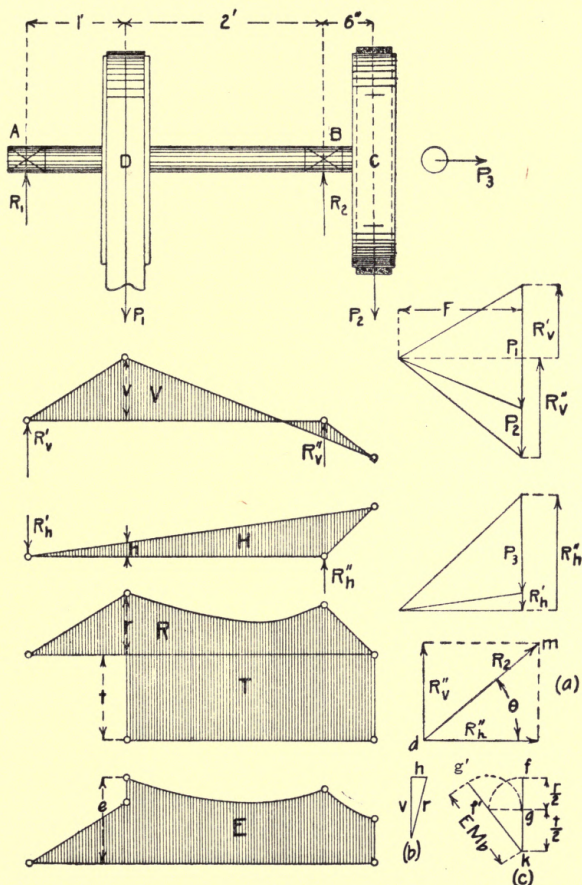


FIG. 19

ported in bearings at $A$ and $B$, and carrying a 26-inch driving pulley $C$ and a 24-inch driven pulley $D$, both of

5-inch face. The belt drive at $C$ is assumed to be horizontal; that at $D$, vertical. Take the vertical load at $D$ as 250 pounds, this including the weights of pulley and belting and the pull of the belt tension. At $C$, let the vertical load, or weights of pulley and belting, be 100 pounds, and the horizontal load, or belt tension, 200 pounds.

The shaft is bent in a vertical plane by a force $P_1 = 250$ pounds acting at $D$ and a force $P_2 = 100$ pounds acting at $C$, and, in a horizontal plane, by the force $P_3 = 200$ pounds acting at $C$. It is also twisted, between $C$ and $D$, by a driving moment $= 200 \times \frac{26}{2} = 2600$ pound-inches.

The diagram for the vertical bending moments is marked $V$. The moments shown by the portion of the diagram above the zero line are positive, *i.e.* the lower side of the shaft is in tension and is bent in a convex curve; the moments below the line are negative, the lower side of the shaft being compressed. The reactions are: $R'_v = 150$ pounds and $R''_v = 200$ pounds.

Diagram $H$ gives the horizontal bending moments. The latter are all positive, and, assuming that the pulley driving $C$ is in front of it, the rear side of the shaft is in tension and convex. The reactions are: $R'_h = 33.3$ pounds and $R''_h = 233.3$ pounds, the two acting in opposite directions.

The linear scale of the original shaft diagram was 1 foot = 1 inch, or 12 to 1; the scale of the force polygon was 200 pounds = 1 inch; and the pole distance $F$ was $1\frac{1}{4}$ inches = 250 pounds. Hence (Art. 12), the

moment scale for $V$ and $H$ was $12 \times 250 = 3000$ pound-inches per inch of measured length of any ordinate.

The actual reactions at the bearings are the resultants of the vertical and horizontal reactions, as above.    Thus, as in ($a$), laying off $R''_v$ and $R''_h$ perpendicular to each other, their resultant $dm = R_2$, inclined at the angle $\theta$ to the horizontal; $R_1$ can be found similarly, but its inclination differs and its line of action lies on the other side of the shaft.

Diagrams $V$ and $H$ are combined to form the resultant bending moment diagram $R$ by a method (Art. 14) similar to that used for the reactions.    Thus, as in ($b$), let $v$ be the vertical bending moment at the section $D$ of the shaft and $h$ the similar horizontal moment; their resultant $r$ is, on the same moment-scale, the corresponding ordinate of diagram $R$.    The latter diagram gives simply the magnitudes of the resultant ordinates at all points, without regard to their sign; it is not a plane surface, but is warped, since the inclination of the ordinates changes continually.

The diagram for the twisting moment between $C$ and $D$ is represented by the rectangle $T$, whose height is made equal to this moment, or 2600 pound-inches, on the same moment scale as before.    Diagrams $R$ and $T$ are combined to form diagram $E$ for the equivalent bending moments by the method shown in ($c$).    The equation for the equivalent bending moment (Art. 14) is :

$$EM_b = \tfrac{1}{2}(M_b + \sqrt{M_b^2 + M_t^2}),$$

in which $M_b$ and $M_t$ are respectively the bending and twist-

ing moments at the section considered and $EM_b$ is the corresponding equivalent bending moment. To apply this expression graphically for any section, as that at $D$, lay off, as in $(c)$, $fg$ and $gk$ equal respectively to one-half the bending and twisting moments. Revolve $f$ to $f'$ and $g$ to $g'$. Then, for the section at $D$:

$$EM_b = g'k = \text{ordinate } e.$$

This follows since, for that section,

$$f'g' = \tfrac{1}{2} M_b \text{ and } f'k = \tfrac{1}{2}\sqrt{M_b{}^2 + M_t{}^2}.$$

This method is used for all sections to the right of $D$; to the left of it, no twisting moment exists and the original bending moment holds.

Considering strength only, the shaft should be designed for the equivalent bending moment shown by the maximum ordinate in diagram $E$. This ordinate is that at section $D$ and is equal to 2516 pound-inches. The resisting moment (Art. 10) of a solid cylinder to bending strain is $SI/c$, in which $S$ is the maximum permissible working stress and $I/c = \pi d^3/32$, $d$ being the diameter in inches. Equating the equivalent bending and resisting moments and taking $S$ as 16,000 pounds per square inch:

$$2516 = 16,000 \times \pi d^3/32,$$
$$d = 1.17 \text{ inches,}$$

which is less than is customary in practice, since the stiffness of the shaft and the bearing surface at the journals must be considered, as well as its strength to resist bending and shearing. The small diameter of

the shaft in this case is produced mainly by the relatively low value of the bending moment, and this low value is due to the fact that the shaft is short and the weight of pulley $C$ acts to balance that of pulley $D$.
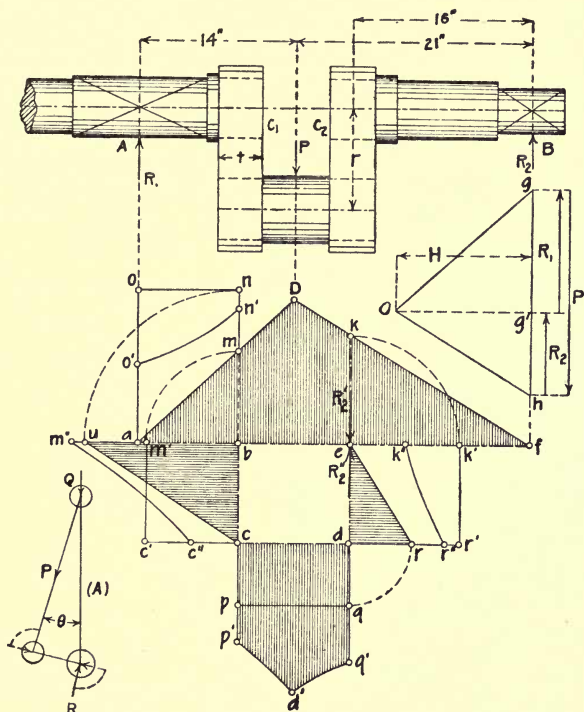


FIG. 20

14. CENTRE CRANK SHAFT: TWISTING AND BENDING COMBINED. Figure 20 represents the centre crank shaft of a vertical, high pressure engine, the shaft being supported in bearings at $A$ and $B$ and delivering its

F

power as a twisting moment at the left-hand end.    For
simplicity, neglect the weight and friction of the shaft
and reciprocating parts.    It is required to find the
equivalent bending moments on the shaft and cranks
when the engine is developing its maximum power, *i.e.*,
when, as in $(A)$, the connecting rod and cranks are at
right angles on the downward stroke.

The data are: cylinder diameter, 15 inches; stroke,
18 inches; length of connecting rod, 29 inches; maxi-
mum unbalanced pressure on piston, 100 pounds per
square inch; force on piston at position shown in
$(A) = Q = \pi \times 15 \times 15 \times \frac{1}{4} \times 100 = 17{,}671$ pounds; cor-
responding thrust on connecting rod $= P = Q/\cos\theta = 17{,}671 \times 1.04 = 18{,}400$ pounds, about.

In the position shown in $(A)$, the thrust $P$ is resisted
by an equal, opposite, and parallel reaction $R$, whose
line of action passes through the centre of the shaft.
To simplify the diagrams to be drawn, the forces $P$ and
$R$, considered as bending forces only, are assumed to be
revolved through 90 degrees, as shown by dotted lines,
and hence to act in the plane of the crank axes—an
assumption which evidently does not alter the magni-
tude of the bending action of these forces on the shaft
and crank pin.    With regard to the bending of the
crank and the twisting of the latter, of the crank pin,
and of a portion of the shaft, the two forces are con-
sidered as acting in their original planes.

Let *abcdef* represent the neutral axes of the shaft,
cranks, and crank pin, *i.e.* a line coinciding with the
axes of the shaft and pin and passing through the

centres of gravity of all transverse sections of the cranks.

The shaft is subjected to bending from $a$ to $b$ and from $e$ to $f$, and to twisting from $b$ to $a$ and onward to the left; the crank pin is under both bending and twisting strains throughout its entire length ; and both cranks are twisted on their neutral axes and are also bent as cantilevers (Art. 7).

On the load-line $gh$, lay off $P$ and draw the force polygon $Ogh$ and the bending moment diagram $aDf$. The ray $Og'$, parallel to the closing line $af$, determines the magnitude of the reactions, which are : $R_1 = 11,040$ pounds and $R_2 = 7360$ pounds. With a linear scale of 8 to 1, a force scale of 8000 pounds per inch, and a pole distance of $1\frac{1}{2}$ inches, the moment scale (Art. 12) is 96,000 pound-inches per inch.

*Shaft.* The right-hand section of the shaft acts simply as a support and is therefore under bending strain only, the moments being given by the portion *fek* of the diagram.

The similar bending moment diagram for the left-hand section is *abm*. This section is also subjected to twisting by the driving moment, $P \times r = 18,400 \times 9 = 165,600$ pound-inches. Using the same moment scale as above, the twisting moment diagram is *bnoa*. Combining the two diagrams by the method employed in Example 13, the equivalent bending moment diagram is $bn'o'a$.

*Crank Pin.* The crank pin and the parts of the cranks lying between the two neutral axes of the latter

are under bending strain, the moments of which are given by the portion *bmDke* of the bending moment diagram.

The pin is also twisted by a moment,

$$R_2 \times r = 7360 \times 9 = 66,240 \text{ pound-inches.}$$

This follows since if, in the original plane of the right reaction $R_2$, there be applied at $e$ two equal and opposite forces $R_2'$ and $R_2''$, each equal to $R_2$, equilibrium will still exist, but the forces $R_2'$ and $R_2$ will form a couple of arm $ef$ tending to twist the crank on its neutral axis, and the force $R_2'' = R_2$ will act, with the leverage $ed = r$, to twist the crank pin. These forces are shown in the diagram as if they were in the plane of the paper; they really act in the plane of $R$ as in ($A$).

The twisting moment diagram is *cpqd*. Combining this with diagram *bmDke*, we have the equivalent bending moment diagram *cp'd'q'd*. Since the pin is cylindrical, the maximum ordinate of the latter diagram, that at the centre, is the only one used in designing the pin. The equivalent bending moment corresponding with this ordinate is 159,360 pound-inches.

*Crank $C_2$.* As has been shown, the right-hand crank is subjected to twisting on its neutral axis *ed* and to bending as a cantilever.

The bending is due to the fact that the crank pin is twisted, through the medium of the crank, by the force $R_2$ acting at $e$. The bending moment at the outer end $d$ of the crank is therefore $R_2 \times r =$ twisting moment

on crank = 66,240 pound-inches; at the inner end $e$, the bending moment is zero. Revolving $dq$ to $dr$, the bending moment diagram for the crank is $edr$, the ordinates being horizontal.

As explained previously, the crank is twisted on its neutral axis by a couple of force $R_2$ and of arm $ef$. The twisting moment is therefore $R_2 \times ef$. Considering the right reaction and not the left as previously, the bending moment $ek$ at the section $e$ is equal to the product of the right reaction by the distance from $e$ to the right support, or $R_2 \times ef$, which is also the twisting moment as above. Revolving $ek$ to $ek'$, the twisting moment diagram is the rectangle $ek'r'd$. Combining the two diagrams, we have the equivalent bending moment diagram $ek''r''d$. The maximum ordinate $dr''$ of this diagram, measured by the moment scale, gives an equivalent bending moment of 100,800 pound-inches.

*Crank* $C_1$. The left-hand crank $C_1$ is subjected to similar bending and twisting actions.

With regard to bending strain, the crank is a cantilever, fixed at $b$ and carrying the load $P$ at $c$. The bending moment at the inner end $b$ is therefore $P \times r = 165,600$ pound-inches, which is the twisting moment $bn$ on the left-hand section of the shaft; the moment at the inner end $c$ is zero. Revolving $bn$ to $bu$, the bending moment diagram is $cbu$, the ordinates being horizontal.

By the same reasoning as that followed for the right-hand crank, it can be shown that crank $C_1$ is twisted on its neutral axis $bc$ by the left reaction $R_1$ acting with the leverage $ab$. The twisting moment is therefore

$R_1 \times ab$, which moment is equal to $bm$, the bending moment on the shaft at the section $b$. Revolving $bm$ to $bm'$, the twisting moment diagram is the rectangle $bm'c'c$.

The combination of the two diagrams gives the equivalent bending moment diagram $bm''c''c$. The maximum ordinate $bm''$ of this diagram corresponds with a bending moment of 180,000 pound-inches.

*Diameter of Crank Pin.* The maximum equivalent bending moment on the crank pin has been found to be 159,360 pound-inches, which is the value of $M$ in the formula, $M = SI/c$. If the pin be a solid cylinder of diameter $d_1$, the rectangular moment of inertia $I = \pi d_1^4/64$ and $c = d_1/2$. Taking a suitable value for the maximum stress $S$, the value of $d_1$ can be determined as in Example 13.

If the crank pin be of steel and made hollow for lightness, the method is the same, except that the value of $I$ is changed, being now the difference between the moments of inertia of the inner and outer circular sections. For example, if the external diameter be $d_1$ and the internal diameter $d_2$, the value of $I$ is $\pi(d_1^4 - d_2^4)/64$ and the area of the cross-section of the pin is $A = \pi(d_1^2 - d_2^2)/4$. Dividing:

$$\frac{I}{A} = \frac{d_1^2 + d_2^2}{16}.$$

$$I = \frac{A}{16}(d_1^2 + d_2^2).$$

Making $d_2 = d_1/2 = c$ and $S = 10,000$ pounds per square inch,

$$A = \tfrac{3}{16}\pi d_1^2.$$

$$\frac{I}{c} = \frac{15}{512}\pi d_1^3 = \frac{159,360}{10,000}.$$

$$d_1 = 5.6 \text{ inches, about.}$$

The maximum shearing stress $S_s$ on the pin can be ascertained from the formula $R_2 \times r = S_s J/c$ (Example 13), in which the twisting moment, $R_2 \times r = 66,240$ pound-inches and the polar moment of inertia, $J = A(d_1^2 + d_2^2)/8$. With $d_1 = 6$ inches and $d_2 = c = 3$ inches, $J = 119$ and $S_s$ is 1670 pounds per square inch, a stress which is immaterial. The diameter of the shaft can be found similarly; its twisting moment is greater, and its maximum equivalent bending moment less, than those of the crank pin.

With steel having an elastic limit of 50,000 pounds per square inch, the factor of safety for the crank pin is $50,000/10,000 = 5$. The diameters as computed, 6 and 3 inches, external and internal, would serve for both a hollow shaft and a hollow pin, if the shaft, cranks, and pin were forged in one piece. If the crank shaft be built up, keys would be required for both pin and shaft, and the external diameters should be increased to allow for the keyways.

*Dimensions of Cranks.* The cranks should be duplicates and should be designed for the maximum moments, which are given by $bm''$ and $dr''$, at the inner and outer ends, respectively. As a rule, the cross-section is rectangular and the thickness $t$ of the web is constant. For uniform strength throughout, the transverse width

$w$ should vary with the maximum moments, as given by either of the two equivalent bending moment diagrams — a refinement which is possible only with a casting.

The hubs on crank pin and shaft should be long enough for adequate keying and to prevent the crank from jarring loose. In built up shafts, this factor usually determines the thickness $t$, which is made uniform throughout and equal to the length of the hub, while the width $w$ is also uniform and equal to the maximum external diameter required for the larger of the two hubs. In any event, the maximum stress at any section of a crank of rectangular cross-section can be determined by proper substitution in the general formula, $M = SI/c$. $M$ is here the maximum bending moment at that section, as given by the equivalent bending moment diagram; $S$ is the corresponding maximum stress which occurs at either of the outer ends of the width $w$; $I = tw^3/12$; and $c = w/2$. The values for $t$ and $w$ are, in each case, taken from the dimensions of the section under consideration.

# CHAPTER IV

## LIVE LOADS : SHEARS AND MOMENTS

The shear and bending moment diagrams which have been considered thus far represent these functions for all sections of a beam carrying *stationary* loads only. This limitation applies also when these diagrams have been constructed for the moving parts of machines, since the members in motion have been assumed to be at rest for an instant and in momentary equilibrium under the action of the driving, resisting, and supporting forces ; and, further, the shears and moments which have been thus determined, refer only to the given position of the moving parts.

While the general effects of a *live* or *travelling* load — as a train passing over a bridge — are, at any instant, precisely the same as those of an equivalent *stationary* or *dead* load in the same position, there is, so far as the maximum stresses in the members are concerned, an important difference between the two cases, in that, with every change in the location of the moving load there are corresponding changes in the shears and moments which that load produces. Hence, in each member of a structure traversed by a moving load, there is a continual variation of stress, the maximum range of

which must be determined in order that the member shall be proportioned for adequate strength under all conditions of service. These variations in the shears, moments, and their corresponding stresses will now be examined.

**15. Variation of Live Load Shear at Sections to the Left of the Load.** (*a*) *Concentrated Loads*. Let Fig. 21 represent a simple beam of span *s*, traversed by a
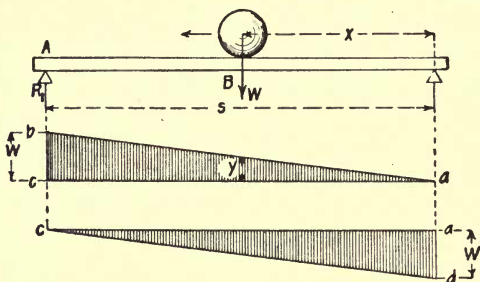


FIG. 21

single concentrated load $W$. The vertical shear $V$ (Art. 8) is the resultant or algebraic sum of all forces to the left of the section considered. With a single moving load, as in this case, $V$ is therefore equal to the left reaction $R_1$. Assume that the load is moving from right to left, and let $x$ be the distance from the right support. Taking moments about the right support:

$$W \times x = R_1 \times s,$$
$$Wx/s = R_1 = V,$$

which is the vertical shear for all sections between the points $A$ and $B$, and to the left of the load.

This expression is the equation of a straight line $ab$ inclined to the horizontal by the angle whose tangent is $W/s$. When $x = s$, $R_1 = V = W$; when $x = 0$, $R_1 = V = 0$. Therefore, the horizontal line $ac$ is the zero line, and, at any position of the load, the ordinate $y$ just below the load — or, strictly speaking, immediately to the left of its centre of gravity — and included in the diagram $abc$ will give $V$ for that section, and similarly for all sections to the left of $W$. It is evident that this is true, whether the load be moving to the right or left, since, in this case, we have but one concentrated load to consider. The diagram $abc$, therefore, represents the variation of the vertical shear at all sections to the left of the load.

For sections to the right of the load, $V = R_1 - W$, an expression which is the same as the preceding equation, except that there is a constant deduction $W$. It is, therefore, the equation of the straight line $cd$, which is parallel to $ba$, and starts at the point $c$ of the zero line. Hence, the diagram $cad$ represents the variation of the shear, now negative, for the sections to the right of the load.

(*b*) *Uniform Load.* Now, consider a moving uniform load, long enough to cover the span when located between the two supports of the beam. In practice, the cars of a railroad train, although not the locomotive and its tender, are considered as such a load in passing over a bridge. This case differs from the one just discussed, in that the total load on the beam varies with the length of the segment of the span covered by the load.

Let Fig. 22 represent a simple beam of span $s$, carrying a partial uniform load $W$, composed of unit loads spaced at a unit distance apart. If $x$ be the number of unit distances covered by the load at a given time and
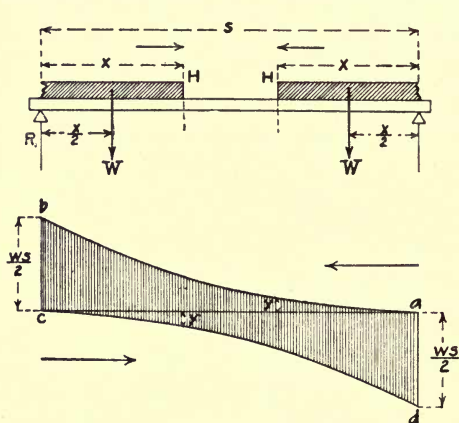


FIG. 22

$w$ be the weight of the unit load, then the total load at that time will be $wx = W$, which is therefore variable. This total load, or resultant of all of the unit loads, will be taken as concentrated at the centre of gravity of the uniform load. It is required to determine the variation in the vertical shear for all sections of the beam to the left of the head of this moving uniform load.

Assume the load to be travelling from right to left. For all sections to the left of the head $H$ of the load, $V = R_1$. Taking moments about the right support:

$$R_1 \times s = W \times x/2 = wx^2/2,$$

$$V = R_1 = \frac{w}{2\,s} \cdot x^2,$$

which is the equation of a parabola. If $x = 0$, $V = 0$; if $x = s$, $V = ws/2$. Plotting $ab$ as a parabolic curve in

accordance with the equation for $V$, the diagram $abc$ represents the variation of $V$, for all sections to the left of $H$, while the load of final length $s$ moves from right to left. Wherever $H$ may be located, the ordinate $y$ immediately below it will give the value of $V$ from $H$ to the left support, for that position of the load.

Again, assume the load to be moving from left to right and measure $x$ from the left support. Taking moments about the right support:

$$R_1 \times s = W(s - x/2) = wx(s - x/2),$$
$$R_1 = wx - wx^2/2s,$$
$$V = R_1 - W = wx - wx^2/2s - wx = -wx^2/2s,$$

which is the same equation as before, except that $V$ is now negative. If $x = 0$, $V = 0$; if $x = s$, $V = -ws/2$. Hence, the diagram $cda$ represents the variation of $V$ for all sections between $H$ and the left support, the ordinate $y$ under $H$ giving the required magnitude of $V$, which is, in all cases, negative.

**16. Influence Diagrams: Influence Lines.** The line $ab$ and the diagram $abc$, Fig. 21, are termed an *influence line* and an *influence diagram*, respectively. In this case, since $V = R_1$, they represent the variation of both the vertical shear and the left reaction. Influence diagrams are usually constructed on the basis of the unit load — a pound, kilogram, or ton. The ordinates of the diagram then refer to the effect of that unit load only, and, to obtain the actual reaction, shear, etc., for the given load, the length of each ordinate is multiplied by the number of pounds, tons, etc., in that load.

## 17. Variation of Live Load Shear at any Given Section of a Beam. (a) *Concentrated Load.* Let Fig. 23 represent a single concentrated load *W*, moving
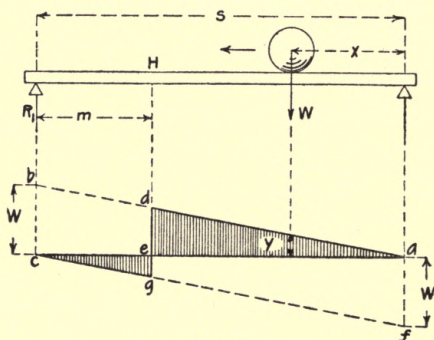


FIG. 23

from right to left over a simple beam of span *s*. It is required to find the variation in the shear at the section *H*, distant *m* units from the left support, while the load traverses the beam.

Assume the load to be to the right of *H* and distant $x$ units from the right support. Taking moments about that support:

$$W \times x = R_1 \times s,$$

$$R_1 = V = \frac{W}{s} \times x,$$

which is the equation of the straight line *ab*, inclined to the horizontal by the angle whose tangent is *W/s*. If $x = 0$, $V = 0$; if $x = s$, $V = W$. Hence, *ac* is the zero line, and the diagram *abc* gives the magnitude of *V* for all sections between *W* and the left support; and, for the section at *H*, the ordinate *y*, immediately under

the load and included within the partial diagram *ade*, represents $V$, while $W$ is to the right of $H$.

When $W$ is between $H$ and the left support, $R_1$ is still equal to $Wx/s$, but $V = R_1 - W$ and is negative, so that the line *cf* is parallel to *ba* and begins at the point *c*, where $x = s$ and $V = 0$. Hence, the ordinate immediately below the load and included within the partial diagram *gec* gives the shear between the load and the left support, which shear is always negative. The complete diagram, representing the variation of $V$ at the section $H$ for all
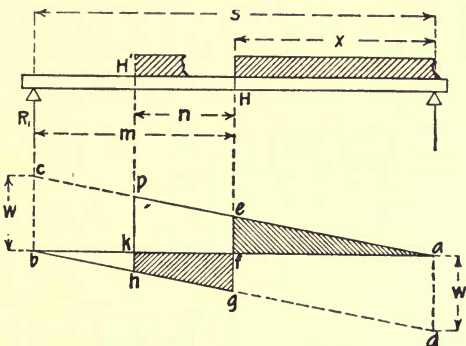


FIG. 24

positions of the load, is *adegce*. This diagram serves whether the load moves toward the right or the left.

(*b*) *Uniform Load.* Let Fig. 24 represent a simple beam of span *s*, traversed from right to left by a uniform load of *w* pounds per unit of length of the span, the load being long enough to cover the span. It is required to find the variation in shear at the section $H$, distant *m* units from the left support, while the head of the load crosses the span.

Draw the zero line $ab = s$; from *b*, erect *bc*, and, from *a*, let fall *ad*, both equal to *w*; draw *ac* and *bd*. Let

$x =$ length of the load, as measured from the right support. Then, as shown previously, while the head of the load is to the right of section $H$, the shear at that section is positive and is:

$$V = R_1 = wx^2/2\ s.$$

With the head of the load at $H$, this expression is the area of the triangle *aef*, forming the part of the diagram below the load, since:

$$ef : x :: w : s,$$
$$ef = wx/2,$$
$$\text{area } aef = ef \times x/2 = wx^2/2\ s = V.$$

When the lead moves to the section $H'$, distant $n$ units from $H$, the shear at section $H$ is the resultant of the positive shear produced by the partial load of length $x - n$ ($x$ being now the distance from the right support to the point $H'$) and the negative shear due to the load of length $n$, both in the positions shown. This resultant shear is the difference between the areas of the positive and negative sections of the diagram below the load, or area *aef* minus *fghk*. As before, $R_1 = wx^2/2\ s$, but the shear at section $H$ is now equal to $R_1$ minus the load $nw$ to the left of $H$, or

$$V = R_1 - nw = wx^2/2\ s - nw.$$

With the head of the load at $H'$, $wx^2/2\ s$ is the area of the triangle *akp*, while $nw$ is the area of the parallelogram *eghp*. Hence

$$\text{area } akp - \text{area } nw = \text{area } aef - \text{area } fghk.$$

**18. Maximum Live Load Shear.** From Fig. 24, it will be seen that, when the given section, as that at $H'$, divides

the span into two unequal segments, the maximum numer-
ical value of the live load shear due to a uniform load will
occur when the load covers fully and only the greater
segment of the span, and that this value will grow with
increase in the length of the segment, as compared with
that of the span.    If this greater and loaded segment lies
to the right of the section considered, the maximum live
load shear will be positive; if to the left, negative.

With a series of concentrated loads differing in mag-
nitude and unequally spaced, the principle as to the
loading of the greater segment of the span to obtain the
maximum live load shear still holds, except that the maxi-
mum shear may occur when one or more of the loads
have passed beyond the section considered and are sup-
ported by the smaller segment of the beam.    This fol-
lows from the fact that the value of the shear depends
primarily on that of the left reaction, and that the value
of the latter grows with every added load borne by the
beam.    Hence, with some of the loads on the lesser
segment, the shear, or algebraic sum of the increased
reaction and these loads, may be greater than the reac-
tion only with the smaller number of loads which the
length of the larger segment will admit.    The final
shear at any section is, in any case, the algebraic sum
of those due to the dead and live loads.

**19. Counterbracing.**    The upper diagram, Fig. 25, is
an outline of a bridge truss* of the Pratt type, in which

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* Bridge trusses are built either as *deck* or *through* spans.    In the former,
the roadway rests on the upper chord; in the latter, on the lower chord.

the diagonal members are built to withstand tension only. As shown in Fig. 24, a uniform load, like a train of cars crossing a bridge truss, produces positive shear in moving from the right abutment to the left and nega-



FIG. 25

tive shear in passing in the other direction. The dead load shear is positive from the left abutment to the middle of the truss and negative from the middle to the right abutment. With the train coming from either direction and pass-

ing over a 5-panel truss, like that shown in Fig. 25, the resultant of the shears due to dead and live loads is always positive in the first and second panels (counting from the left), and always negative in the fourth and fifth panels; but, in the middle panel, this resultant shear is positive when the train comes from the right and negative when it enters from the left.
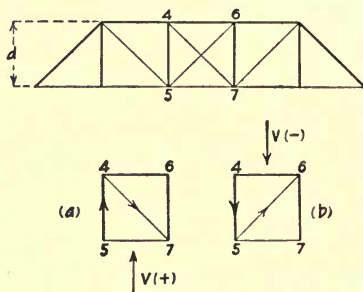
The vertical shear which exists in a panel is a force which acts upward if the shear be positive, and downward if it be negative. In the Pratt truss, under any system of loading, the vertical members are always in compression, and the diagonals are built to take tension only. Hence, when the shear in the middle panel changes from positive to negative, or the reverse, there must be a corresponding shifting, from one diagonal to

the other, of the tensile stress whose vertical component is to resist this shear, *i.e.* the main tie (diagonal) ceases to act and the counter-tie takes the load, or *vice versa*, the two ties being oppositely inclined.

Thus, in (*a*), Fig. 25, the shear in the middle panel is positive; the stress in the vertical *5–4* is compressive, and therefore acts toward panel-point *4*; and the stress in the main diagonal *4–7* is tensile, and acts from point *4*. The panel is subjected to the upward force *V*, and the only vertical forces at point *4* are the compressive stress *5–4* and the vertical component of the tensile stress 4–7. For equilibrium, these vertical forces must be equal and opposite. Hence, *V* is equal in magnitude to stress *4–5*, and also to the vertical component of the stress *4–7*, and is the same in direction as stress *5–4* with regard to panel-point *4*. When, as in (*b*), the shear in the panel becomes negative, *V* is equal and opposed to the vertical component of the stress in the counter-tie *5–6*, and is equal and like the stress *4–5* in the vertical, both with regard to panel-point *5*.

When the main diagonal is acting, the counter-tie buckles and is not under stress; owing to their opposite inclination, both ties cannot be strained simultaneously. This is the fundamental principle of *counterbracing*, which is applied whenever a member is subject to stress reversal and is fitted to take tension only. In the Warren truss, the members are built for both stresses, and counterbraces are not required. In roof trusses the wind loads may cause a reversal of stress in some members, thus necessitating counterbracing.

**20. Variation of Bending Moment at any Given Section of a Beam.** (*a*) *Concentrated Loads*. Let Fig. 26 represent a simple beam of span *s*, traversed from right to left by a single concentrated load *W*. It is required to find the variation in the bending moment at the section
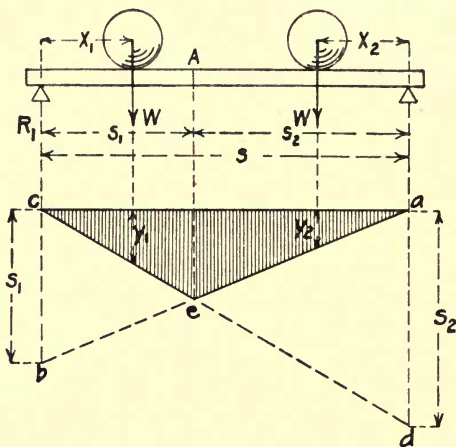


FIG. 26

*A*, distant $s_1$ from the left support and $s_2$ from that at the right, while the load crosses the beam.

The bending moment (Art. 9) at any section is the moment of the left reaction about any point in that section, minus the similar moments of the loads, if any, to the left of the section. First, let the load *W* be between the section *A* and the right support, at the distance $x_2$ from the latter. The left reaction is $R_1 = Wx_2/s$, and the bending moment at section *A* is:

$$M_2 = R_1 \times s_1 = Ws_1 \times x_2/s,$$

which is the equation of the straight line $ab$, inclined to the horizontal by the angle whose tangent is $s_1/s$, the weight $W$ being taken, for convenience, as a unit load.

Now, assume the load $W$ to be between the section $A$ and the left support, at the distance $x_1$ from the latter. Then, $R_1 = W(s - x_1)/s$, and the bending moment at section $A$ is:

$$M_1 = R_1 \times s_1 - W(s_1 - x_1) = Ws_2x_1/s.$$

Taking $W$ as the unit load, this expression is the equation of the straight line $cd$, inclined to the horizontal by the angle whose tangent is $s_2/s$.

The triangle $aec$ is the influence diagram for the bending moments at section $A$, when $W$ is equal to the unitload, and the ordinate $y_1$ or $y_2$ immediately below the load gives the bending moment at $A$ for that position of the load. This follows, since:

$$y_1 : s_2 :: x_1 : s,$$
$$y_1 = s_2 \times x_1/s = M_1;$$

and

$$y_2 : s_1 :: x_2 : s,$$
$$y_2 = s_1 \times x_2/s = M_2.$$

The diagram has been constructed for the unit load. To obtain the bending moment for a load $W$, greater than unity, each ordinate should be multiplied by the number of units in $W$.

(*b*) *Uniform Load*. The influence diagram in **Fig.** 26 is drawn for the unit load. With a total uniform load $W$ covering a segment or all of the span, there is a series of unit loads, each equal to $w$, spaced at unit

distance apart, over the segment covered. The bending moment due to each unit load is given by the ordinate immediately below that load. Hence, the total bending moment produced by the total load $W$ is the sum of these unit moments, or the area of the portion of the influence diagram below the total load $W$.

Thus, in Fig. 26, let the uniform load be moving from right to left and let the head of the load be at the distance $x_2$ from the right support. Taking moments about that support:

$$R_1 \times s_2 = wx_2 \times x_2/2 ;$$

and the left reaction is:

$$R_1 = wx_2^2/2\, s.$$

The bending moment at the section $A$ due to the total load $W = wx$, is:

$$R_1 \times s_1 = ws_1x_2^2/2\, s = M.$$

But:

$$y_2 : s_1 :: x_2 : s,$$

$$y_2 = s_1x_2/s ;$$

and the area of the triangle below the load is:

$$y_2 \times x_2/2 = s_1x_2^2/2\, s = M,$$

since $w$, as the unit load, is equal to unity.

### 21. Maximum Bending Moments due to Live Loads.
The maximum bending moment which is possible at any given section of a beam under any system of loading, depends on two conditions: first, the value of the moment rises with increase in the magnitude of the left reaction; and, second, this value is lessened by increase

in the moments of any loads to the left of the section. Hence :

(*a*) With uniform live loads, the maximum moment at any section of a beam occurs when the load covers the entire span.

(*b*) With concentrated live loads, unequal in magnitude and unequally spaced, the maximum moment at any section occurs when the span carries the greatest possible load, consistent with having one of the largest loads at or near the given section.

**22. Live Load Stresses in Trusses and Plate Girders.** Either a truss or a girder, under the action of live loads, is essentially but a beam, and as such is subjected to bending moments and vertical shear.

In a truss such as that shown in Fig. 25, the primary function of the web members — as the vertical *4-5* and the diagonal *4-7* — is to resist the vertical shear, since, if the chords be parallel, these web members are the only ones having either vertical stresses or stresses which can be resolved vertically. Hence, the vertical component of the stress in any diagonal of this truss is equal in magnitude to the vertical shear in the panel in which the diagonal is located, and the stress in the vertical meeting a diagonal at a panel-point in the unloaded chord is equal in magnitude and opposite in direction to the vertical component of the stress in the diagonal.

The chord members — as *4-6* and *5-7*, Fig. 25 — take the tensile and compressive stresses, respectively,

produced by the bending moments.   The stress in either
chord, at any given section of such a truss, is equal to
the quotient of the bending moment at that section,
divided by the depth of the truss.   Thus, Fig. 27 repre-
sents a segment of a Pratt truss and a corresponding



FIG. 27

section of the equilibrium
polygon constructed for
the loads and reactions, as
for a simple beam.   Ap-
plying Rankine's Method
of Sections, let the truss
be cut on the line $ab$, im-
mediately to the right of
the vertical $2$–$3$, so that
the bending moments at
$ab$ and $2$–$3$ will be virtu-
ally the same.   To main-

tain equilibrium, apply to the severed members, the forces
$F$, $F_1$, and $F_2$, each of which is equal to and like the stress
in the member to which it is applied.   The segment of
the truss is then in equilibrium under the action of these
applied forces, the load $P$ at panel-point $3$, and the left
reaction $R_1$.   The two latter can be replaced (Art. 11)
by their resultant $r$, acting vertically through the point
$E$, the intersection of the prolonged sides $BC$ and $AD$ of
the equilibrium polygon.   For equilibrium to exist, the
algebraic sum of the moments of $r$ and the three applied
forces, about an axis perpendicular to their plane of action,
must be zero.   Let $d$ be the depth of the truss.   Then,
taking moments about panel-point $2$, where the lines of

action of $F_1$ and $F_2$ meet and their moments are zero, we have:

$$F \times d = r \times l$$
$$F = rl/d = M/d.$$

since, by Art. 11, $r \times l$ is equal to the bending moment at the section $ab$ or $2$–$3$. It will be observed that these principles relating to the web- and chord-stresses, apply only to trusses with parallel chords. When the upper chord is 'broken' and thus inclined to the lower, there is a vertical component of its stress which must be considered.

In a plate girder, the web plate is usually assumed to take only the same stresses as the web members of a truss with parallel chords, and the flanges — composed of angles and cover-plates, if any — to act as the chord members. The customary method of design regards the web plate as resisting vertical shear only and the flanges as taking the full bending stress; in more accurate work, the resistance of the web plate to bending is considered.

### EXAMPLES

15. PLATE GIRDER BRIDGE WITH LOCOMOTIVE WHEEL LOADS: MAXIMUM MOMENTS AND SHEARS. The weight of a locomotive, as distributed among its driving and pilot wheels, forms a system of vertical loads, more or less unequal and unequally spaced. In designing a bridge, the maximum moments and shears must be determined for the greatest live load which will pass over it. For simplicity in the following example, the locomotive only will be considered; it is usual to design the

bridge for two coupled locomotives with their tenders, followed by a uniform train load.

Let $AB$, Fig. 28, represent a deck, plate girder bridge for a single track railroad, the effective span and depth
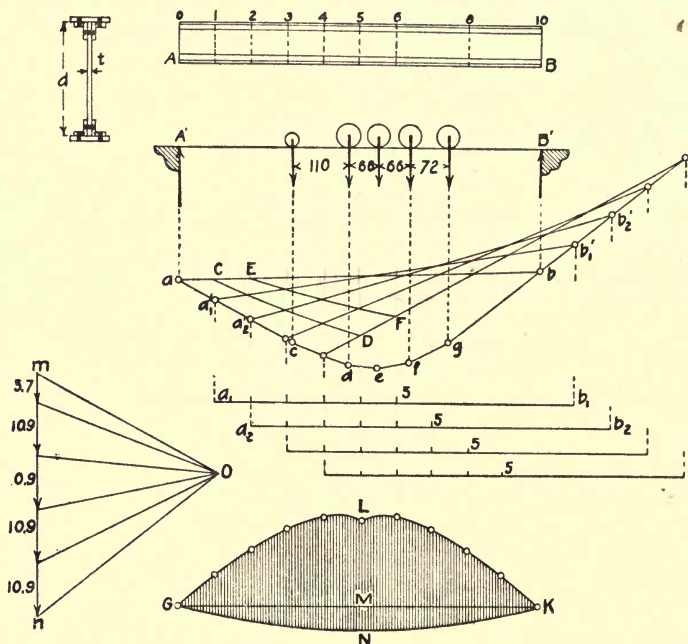


FIG. 28

of the girders being 60 feet and 6 feet, respectively. The load carried by the bridge is one Northern Pacific tandem compound locomotive (1901), whose total weight is 198,000 pounds, 175,000 pounds being on the driving wheels and 23,000 pounds on the engine truck. These weights, when distributed as shown in the figure, are, in

tons of 2000 pounds, 10.94 tons on each driver and 5.75 tons on each pilot wheel. The weights on the wheels at one side of the locomotive constitute the system of live loads borne by each of the two girders forming the bridge. The spacing of these loads is marked in inches below the line $A'B'$ indicating the span.

(*a*) *Maximum Moments*. The system of loads may be located in any position on the span, although preferably the first driving wheel should be near the middle of the girder, as in the figure. The moments and shears should be found at sections reasonably close together throughout the span. Therefore, divide the girder into any number of segments, say 10, at intervals of 6 feet.

On the load line *mn*, lay off the loads and draw the force polygon *Omn*, with pole distance equal to 5 times the depth, or 30 feet, since, by using thus a multiple of the depth, the moment diagram is, by Art. 22, available for the measurement of flange stresses. Construct the corresponding equilibrium polygon *acdefgb*. Then, the bending moment at any one of the numbered sections of the girder will be given by the length of the ordinate immediately below it and between the closing line *ab* and the perimeter $a \cdots e \cdots b$ of the equilibrium polygon, this length being measured by the proper moment scale (Art. 12).

Now, shift the span one interval to the right, as at $a_1b_1$; prolong the side *gb*; project $a_1$ and $b_1$ to $a_1'$ and $b_1'$, respectively, and draw the new closing line $a_1'b_1'$. This operation is, in effect, moving the system of loads a distance equal to one interval to the left on the span

as originally placed, and the equilibrium polygon for this new location is $a_1' \cdots e \cdots b_1'$. The bending moment at any section of the girder is then given by the length of the corresponding ordinate above that section (as marked on the line $a_1 b_1$), and included between the perimeter and the closing line $a_1' b_1'$.

The span is then shifted successively to several new positions to the left, as $a_2 b_2$, etc., each position differing from the one immediately preceding it by the space of one interval; and the corresponding equilibrium polygon is constructed in each case. This process is continued until every section, from the middle of the girder to the left abutment, has been subjected to its maximum bending moment. Each numbered section from 1 to 5, in the various locations of the span, is then projected upon the corresponding closing line of the equilibrium polygon, and a free curve, known as the *stress curve*, is drawn through the points thus determined. Thus, $CD$ and $EF$ are the stress curves for sections 1 and 2, respectively. The maximum bending moment at section 1 is evidently given by the length of the greatest ordinate included between the stress curve $CD$ and the perimeter $a \cdots e \cdots b$. The method for all other sections is similar.

The maximum moments thus obtained for the live load are plotted upward from the axis $GK$, for sections 1 to 5, giving points on the semi-curve of maximum moments $GL$; the full curve $GLK$ is symmetrical about its centre line $LM$, and is cusped at $L$, as shown.

The *dead load*, *i.e.*, the total weight of one girder, of

one-half the lateral bracing and cross-framing, and of the floor system, may be taken, in this case, roughly, as about 600 pounds per linear foot of the span, or 18 tons for each girder. This load is considered to be uniformly distributed over the span, and the moments due to it are represented by the curve *GNK*, which is drawn, for convenience, below the axis *GK*, to the same moment scale as before. The maximum moment at any section, due to the dead and live loads, will then be given by the length of the corresponding ordinate included between the two curves. The curve *GNK* is a parabola whose equation (Art. 11) is:

$$M = wsx/2 - wx^2/2,$$

in which *M* is the bending moment in ton-feet at the section distant *x* feet from the left support, *w* is the uniform load in tons per lineal foot of the span, and *s* is the span in feet.

(*b*) *Maximum Shears*. It can be shown by computation or by measurement from the diagrams that the maximum live load shear at any section occurs when the first driving wheel is immediately above that section, the shear being then equal to the left reaction, less the load of 5.75 tons on the pilot wheel. The maximum shear can hence be found from a diagram similar to that in Fig. 28, by successively locating each section of the span below the first driver, constructing the corresponding equilibrium polygon, and then (Art. 11) drawing a ray in the force polygon parallel to the closing line of the equilibrium polygon. The distance between the in-

tersection of this ray with the load line *mn* and the point *m* will then be the left reaction for that position of the loads, and, deducting from this the first load of 5.75 tons
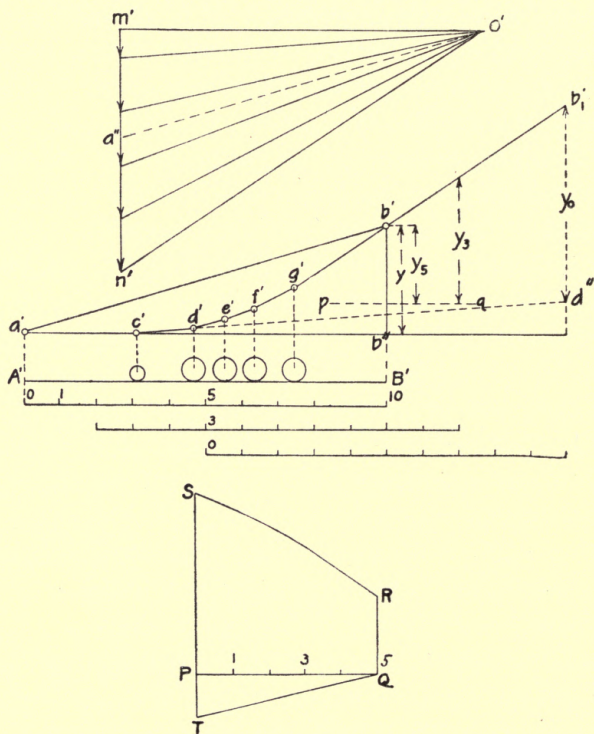


FIG. 29

on the pilot wheel, we have the maximum live load shear at the section immediately under the first driving wheel.

A more convenient method is shown in Fig 29. A new force polygon $O'm'n'$ is constructed, having the pole $O'$ at a horizontal distance from $m'$ equal to the span.

Then, assuming the loads to be located as on the span
$A'B'$, Fig. 28, the corresponding equilibrium polygon will
be $a'c'd'e'f'g'b'$. In the force polygon, draw the ray
$O'a''$ parallel to the closing line $b'a'$. Since the tri-
angles $O'm'a''$ and $b''a'b'$ are equal in all respects, $b'b''$ is
equal to $a''m'$, and hence to the left reaction for this
position of the loads. This principle is general, applying
to all other values of the left reaction, when found
similarly.

Prolong the sides $g'b'$ and $a'c'$, and draw the indefinite
line $pq$ parallel to $a'c'$, and at a vertical distance from it
equal to the pilot load, 5.75 tons. Lay out the span
with section 5 immediately below the first driving
wheel; the ordinary $y$ above the right end of the span is
the left reaction for this position of the loads, and the
ordinate $y_5$ is the maximum live load shear at section 5
since the pilot load must be deducted. Similarly, $y_3$ is
the similar shear at section 3, and so on. When the
first driver reaches the left support, its line of action co-
incides with that of the left reaction, and the pilot load
is no longer on the girder. Hence, the side $c'd'$ is pro-
longed to $d''$, and the live load shear at section $o$ is
equal to the left reaction, and is represented by the
ordinate $y_0$. In this method, it is assumed that the given
sections — as 5, 3, and $o$ — are successively located im-
mediately below the first driving wheel, which is only
approximately the position shown in Fig. 29, the latter
position being, for comparison, the same as that in Fig.
28.

As stated previously, the live load shear is positive

when the load moves from right to left. Laying off these shears from the axis $PQ$ upward, $PQRS$ becomes the diagram of live load shears from the left support to the middle of the girder. The total dead load on each girder is 18 tons. As this load is uniformly distributed, the dead load shear is one-half of it, or 9 tons, at the left support, and decreases uniformly to zero at the middle (Art. 8). This shear is also positive, but, for convenience, is laid off below the axis in the diagram $TPQ$. The final shear at any section is then represented by the corresponding ordinate in the combined diagram $STQR$. The diagram for the right half of the girder, and with the load moving from left to right, will be similar but reversed, corresponding sections having shears of the same magnitude, but both those due to dead and live loads being negative.

(*c*) *Flange Stresses: Impact.* By Art. 11, the bending moment at any section of a beam is equal to the product of the corresponding ordinate in the equilibrium polygon by the pole distance $H$ of the force polygon. By Art 22, the flange stress, which corresponds with the chord stress in a truss, is equal to the bending moment, divided by the depth of the truss or girder. In this case, the pole distance in the first force polygon was taken as 5 times this depth. Hence:

$$M = y \times H,$$
$$H = 5\,d,$$
$$A \times S = M/d = Hy/d = 5\,y,$$

in which $M$ is the bending moment at the given section, $y$ is the corresponding ordinate of the bending moment diagram, $p$ is the depth of the girder, $A$ is the total area

of either flange, and $S$ is the flange stress, per square inch, at the given section.    Hence, the moment diagram $GNKL$, Fig. 28, will serve as a total $(A \times S)$ flange stress diagram, if its ordinates be measured by a force scale 5 times greater than that used for the force polygon, *i.e.*, if the scale of $Omn$ be 20 tons to the inch, the flange stress scale will be 100 tons to the inch.    The unit stress at any given section of the web, due to the vertical shear, can be found by dividing the total shear, as given by the corresponding ordinate of the diagram $STQR$, Fig. 29, by the cross-sectional area of the web.

In bridges, the stresses, as determined above, are increased by an allowance for the *impact* resulting from the sudden application of the live load.    Formulas for this allowance are given in works on this subject; the increase is usually about 70 per cent.

(*d*) *Summary*.    Tabulating the results, as measured from the diagrams by the moment scale, we have, neglecting the allowance for impact :

| Section | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Bending moment, dead load, ton-feet | o | 48.6 | 86.4 | 113.4 | 129.6 | 135. |
| Maximum moment, live load, ton-feet . | o | 198.3 | 354. | 498.5 | 558. | 588.5 |
| Total bending moment | o | 246.9 | 440.4 | 611.9 | 687.6 | 723.5 |
| Vertical shear, dead load    .    .    . | 9 | 7.2 | 5.4 | 3.6 | 1.8 | o |
| Maximum live load shear, tons    .    .    . | 37.5 | 33.2 | 28.5 | 23.5 | 18.6 | 13.7 |
| Vertical shear, total, tons    .    .    .    .    . | 46.5 | 40.4 | 33.9 | 27.1 | 20.4 | 13.7 |

H

16. WARREN GIRDER FOR OVERHEAD CRANE: MAXIMUM STRESSES. Girders for overhead cranes are of several types. The rolled beam, the plate girder, the box girder, and the truss in different forms have each its place for this service. The selection depends mainly on the span and the maximum live load to be transported, although economy in weight and construction are primary requisites.

The upper diagram, Fig. 30, is the elevation in out-
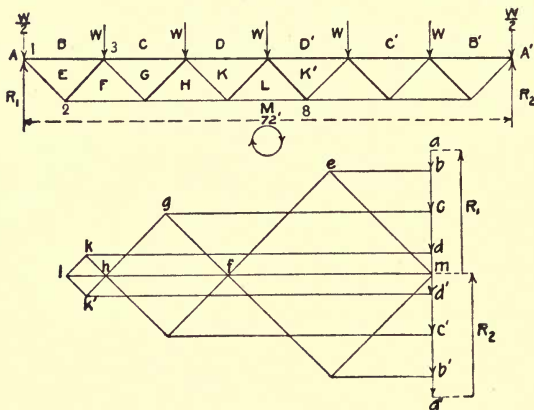


FIG. 30

line of a Warren girder or truss with parallel chords, as employed in such cranes. In this girder, the web members are usually all diagonals and they are built to withstand both tension and compression; verticals may be inserted as struts between each joint of the lower chord and the middle of the panel above it, when the length of a panel is considerable. The crab, which carries the live load, has four wheels, tracking on the upper

chords of the two parallel girders.   The conditions are thus, in general, the same as those of Example 15, except that the plate girder has been replaced, in this case, by the Warren truss.   Let the girder, Fig. 30, be a 6-panel, deck truss of 72 feet span and 6 feet in depth. Consider first:

(a) *Dead Load Stresses*.   The dead load on each girder consists of the weight of one girder, its cross-shafting, and platform.   Take it as 6 tons and consider it as uniformly distributed over the girder.   Then, as in the roof truss, Example 7, there will be a weight $W$ = 1 ton, acting at each of the inner joints of the upper chord, and a weight $W/2 = \frac{1}{2}$ ton, on each of the end joints.   Since the load is uniformly distributed, the reactions $R_1$ and $R_2$ are equal and each is 3 tons.

Lay off the loads *ab*, *bc*, etc., on the load-line *aa'*. Bisect this line at *m*.   Then, $ma = R_1$ and $a'm = R_2$. The closed force polygon for the loads and reactions is then *abcdd'c'b'a'ma*.   From *m*, draw the indefinite horizontal line *ml*.

Joint *1* is in equilibrium under the action of the reaction $R_1$ and the load $W/_2$, which are known, and the stresses in the members $BE$ and $EM$, which stresses are to be determined.   From *b* and *m* lay off *be* and *me*, parallel respectively to $BE$ and $EM$ and meeting at *e*.   Then, taking the loads in clockwise order, the closed force polygon for the joint is *abema*.   The stress *be* thus acts toward the joint and is therefore compressive, while *em* acts from it and is tensile.

At joint *2* there are acting the known stress *me* and

the stresses in $EF$ and $FM$, which are to be determined. From $e$ and $m$, lay off $ef$ and $mf$, parallel to their corresponding members and meeting at $f$. The force polygon is $mefm$, $ef$ is compressive, and $fm$ is tensile.

In a similar way, the force polygon for joints $3$ to $8$, inclusive, are drawn. Thus, for joint $3$, the polygon is $bcgfeb$; for joint $4$, $fghmf$; etc. Since the load is uniformly distributed, the stress diagram is symmetrical about its axis $lm$. It will be seen that the diagonals,
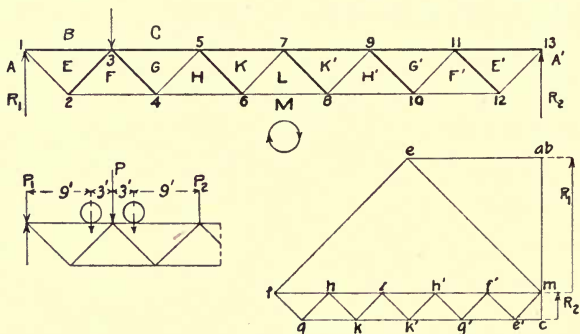


FIG. 31

when under dead load, are alternately struts and ties, the order changing at the middle of the truss.

(*b*) *Live Load Stresses.* The maximum weight to be transported may be taken as 15 tons and the weight of the crab as 5 tons, which will bring a load of $20/4 = 5$ tons on each of the four wheels or runners of the crab. The distance between these runners, measured parallel to the girder, is 6 feet.

The graphic analysis of a truss is based on the assumption (Art. 5) that all loads are transferred to, and

concentrated at, the joints of the truss. Hence, if the centre of the crab be above joint $3$,— as in the partial diagram, Fig. 31, — each runner will be 3 feet from that joint and 9 feet from the joint on the other side of the runner. The load of 5 tons on each runner will then be divided in the proportion of 3 to 1 between the two adjacent joints. In the position shown, there will thus be a load $P = 3.75 \times 2 = 7.5$ tons on joint $3$, a load $P_1 = 1.25$ tons on joint $1$, and a load $P_2 = 1.25$ tons on joint $5$. The crab and the weight transported thus form a symmetrical system of three loads spaced 12 feet apart.

The maximum stresses produced in the members of the truss by the travel of the crab and its load can be found by locating the system of loads centrally above panel-points $3$, $5$, and $7$, in succession, and taking, for each member, the greatest of the three stresses developed in it by these three locations of the system of loads. Since the crab may approach the middle of the girder from either end, it will be necessary to determine the stresses in one-half only of the members.

There are several ways of finding the changes in the stresses due to these changes in the location of the loads : first, the method of the equilibrium polygon can be employed, as in Example 15 ; second, the system can be located at each of the three panel-points in succession, and a separate stress diagram drawn for each case ; and, finally, a single stress diagram, for one load only at one panel-point, can be constructed, and the stresses due to the system at each of the three panel-points can be found by proportion from this stress diagram. This

method of multiples is essentially the use of the influence
diagram in modified form.    While possibly not the most
convenient process in this case, it is given below for in-
formation.

Assume the central load $P = 7.5$ tons to be the only
one on the girder and to be located, as in Fig. 31, at
panel-point $3$.  The reactions are : $R_1 = 6.25$ tons and
$R_2 = 1.25$ tons.   The closed force polygon for the loads
and reactions is *bcmb*.  The stress diagram is constructed
by the same method as that used for the dead load dia-
gram.    The stresses thus determined for the web mem-
bers are tabulated under 'panel-point $3$' in Table 1.

TABLE 1

| WEB MEMBER | CENTRAL LIVE LOAD $P$ AT PANEL-POINT | | | |
| --- | --- | --- | --- | --- |
| | **3** | **5** | **7** | **9** |
| *ME* | + 8.9 | + 7.1 | + 5.3 | + 3.6 |
| *EF* | − 8.9 | − 7.1 | − 5.3 | − 3.6 |
| *FG* | − 1.8 | + 7.1 | + 5.3 | + 3.6 |
| *GH* | + 1.8 | − 7.1 | − 5.3 | − 3.6 |
| *HK* | − 1.8 | − 3.6 | + 5.3 | + 3.6 |
| *KL* | + 1.8 | + 3.6 | − 5.3 | − 3.6 |

The values given in the three remaining columns of
the table are computed from those in the first column.
The method is as follows :

The stress in a diagonal member is directly propor-
tional to the vertical shear in the panel in which the

diagonal is located. Since there is but one load, the
vertical shear is equal to the left reaction. When
the load $P$ is at panel-point $3$, $R_1 = 5/6\,P$ and $R_2 = 1/6$
$P$; when $P$ passes to panel-point $5$, $R_1 = 4/6\,P$ and $R_2$
$= 2/6\,P$. Hence, in the latter case, the stresses in all
diagonals to the left of $P$ are $4/5$ of those in the simi-
lar diagonals in the former case; while, on the right of
$P$, the multiple is $2$ and the stresses are double those
which prevailed previously in similar diagonals. With
$P$ at panel-point $7$, the multiples to left and right of $P$
are, respectively, $3/5$ and $3$; and so on. The signs of
the stresses are determined by the fact that the two
diagonals meeting at the panel-point where $P$ is located
are under compression, and that thereafter, both to left
and right, the diagonals are alternately under tension
and compression. As before, tensile stresses are con-
sidered as positive $(+)$ and compressive stresses as

TABLE 2

| WEB MEMBER | END LOAD $P_1$ OR $P_2$ AT PANEL-POINT | | | |
|:---:|:---:|:---:|:---:|:---:|
| | **3** | **5** | **7** | **9** |
| $ME$ | $+\,1.5$ | $+\,1.2$ | $+\,0.9$ | $+\,0.6$ |
| $EF$ | $-\,1.5$ | $-\,1.2$ | $-\,0.9$ | $-\,0.6$ |
| $FG$ | $-\,0.3$ | $+\,1.2$ | $+\,0.9$ | $+\,0.6$ |
| $GH$ | $+\,0.3$ | $-\,1.2$ | $-\,0.9$ | $-\,0.6$ |
| $HK$ | $-\,0.3$ | $-\,0.6$ | $+\,0.9$ | $+\,0.6$ |
| $KL$ | $+\,0.3$ | $+\,0.6$ | $-\,0.9$ | $-\,0.6$ |

negative $(-)$. The values above the heavy line in the table refer to members to the left of the load $P$.

The load $P_1 = P_2 = 1.25$ tons is equal to $P/6$. Hence, in the same location as $P$, it will develop stresses of the same kind and only one-sixth as large. Dividing the values in Table 1 by 6, we have the stresses in Table 2 for either $P_1$ or $P_2$, as it moves across the girder.

When the system of three loads is centrally above a panel-point, $P$ is at the latter point and $P_1$ and $P_2$ are at adjacent panel-points to the left and right, respectively. Selecting the values for these conditions from Tables 1 and 2, we have, in Table 3, the live load

TABLE 3

| Web Members | ME | EF | FG | GH | HK | KL |
|---|---|---|---|---|---|---|
| System at Panel-point 3: | | | | | | |
| Load $P_1$ at panel-point 1. . | 0 | 0 | 0 | 0 | 0 | 0 |
| Load $P$ at panel-point 3. . | + 8.9 | − 8.9 | − 1.8 | + 1.8 | − 1.8 | + 1.8 |
| Load $P_2$ at panel-point 5. . | + 1 2 | − 1.2 | + 1.2 | − 1.2 | − 0.6 | + 1.6 |
| Stresses due to system  .  .  . | + 10.1 | − 10.1 | − 0.6 | + 0.6 | − 2.4 | + 2.4 |
| System at Panel-point 5: | | | | | | |
| Load $P_1$ at panel-point 3. . | + 1.5 | − 1.5 | − 0.3 | + 0.3 | − 0.3 | + 0.3 |
| Load $P$ at panel-point 5. . | + 7.1 | − 7.1 | + 7.1 | − 7.1 | − 3.6 | + 3.6 |
| Load $P_2$ at panel-point 7. . | + 0 9 | − 0.9 | + 0.9 | − 0.9 | + 0.9 | − 0.9 |
| Stresses due to system  .  .  . | + 9.5 | − 9.5 | + 7.7 | − 7.7 | − 3. | + 3. |
| System at Panel-point 7: | | | | | | |
| Load $P_1$ at panel-point 5. . | + 1.2 | − 1.2 | + 1.2 | − 1.2 | − 0.6 | + 0.6 |
| Load $P$ at panel-point 7. . | + 5.3 | − 5.3 | + 5.3 | − 5.3 | + 5.3 | − 5.3 |
| Load $P_2$ at panel-point 9. . | + 0.6 | − 0.6 | + 0.6 | − 0.6 | + 0.6 | − 0.6 |
| Stresses due to system  .  .  . | + 7.1 | − 7.1 | + 7.1 | − 7.1 | + 5.3 | − 5.3 |

stresses in the web members for the three required positions of the system of loads.

Taking the maximum and minimum live load stresses from Table 3 and combining them with the dead load stresses, as measured from the stress diagram, Fig. 30, we have, in Table 4, the maximum stresses in the

TABLE 4

| Web Members | Stress Range, Live Load | Dead Load Stresses | Maximum Stresses |
|---|---|---|---|
| $ME$ | + 7.1 to + 10.1 | + 3.6 | + 13.7 |
| $EF$ | − 7.1 to − 10.1 | − 3.6 | − 13.7 |
| $FG$ | − 0.6 to + 7.7 | + 2.2 | + 9.9 |
| $GH$ | + 0.6 to − 7.7 | − 2.2 | − 9.9 |
| $HK$ | − 2.4 to + 5.3 | + 0.7 | + 6. |
| $KL$ | + 2.4 to − 5.3 | − 0.7 | − 6. |

web members, as the crab, carrying its live load, travels from the end to the middle of the girder.

17. PRATT TRUSS: UNIFORM LIVE LOAD. The magnitudes of the stresses in the members of a bridge truss, due to dead loads and to uniform live loads, can be determined by stress diagrams, as with a roof truss and with the Warren truss, Fig. 30; these stresses can be found also by the general method of the force and equilibrium polygon. The use of the latter is illustrated in Fig. 32, which represents a Pratt truss, through type, of six panels, having a span of 144 feet and a depth of 24 feet. Let the uniform live load

crossing the bridge be a train weighing 4000 pounds
per lineal foot. One-half of this load will be carried
by each of the two trusses forming a single-track rail-
way, as in this case. This gives a load of 4000/2
= 2000 pounds = 1 ton per lineal foot of span, or a
panel load of 24 tons.
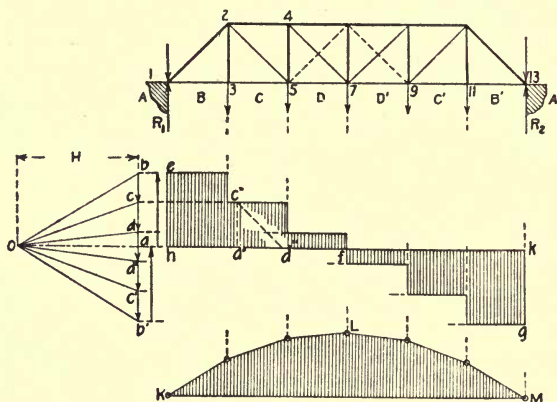


FIG. 32.

Each of the inner panel-points, 3, 5, 7, 9, and 11,
carries this load; the end points, 1 and 13, have each
a half panel load, or 12 tons. As the total load is uni-
formly distributed, the two reactions will be equal.
The total reaction at the left support is 24 × 6/2 = 72
tons; deducting the half panel load at point 1, the
effective reaction is 60 tons.

To any convenient scale, lay off the loads and effec-
tive reactions on the load line $bb'$, and, taking the pole
distance $H$ as a multiple of the depth of the truss, draw
the force polygon $Obb'$ and the corresponding equilib-

rium polygon $KLM$, which polygon is the bending moment diagram for the truss.

Since the vertical shear $V$ is equal to the left reaction, minus any loads to the left of the section considered, the shear diagram $efg$, with zero line $hk$, can be drawn, as shown, by projection from the lower chord of the truss and from the load line of the force polygon. Thus, in the first panel, $V = R_1 = ab$; in the second panel, $V = R_1 -$ load $BC = ab - bc = ac$; etc.

(*a*) *Chord Stresses.* By Art. 22, the total stress in either chord at any given section of the truss is equal to the bending moment at that section, divided by the depth of the truss. In this case, as in Example 15, the pole distance is a multiple of the depth of the truss. Hence, by using a stress scale proportioned for these conditions, the chord stress at any given section can be measured directly from the moment diagram $KLM$. Thus, at the middle of the second panel, the total stress in either chord is 78 tons; at the middle of the third panel, 102 tons; etc. These stresses are compressive in the upper chord and tensile in the lower.

(*b*) *Stresses in Diagonals.* By Art. 22, the vertical component of the stress in any diagonal member of a truss with parallel chords is equal to the vertical shear in the panel in which the diagonal is located. Thus, $a''c'' = 36$ tons is the vertical shear in the second panel and is also the vertical component of $c''a''$, drawn parallel to the diagonal *2-5*. The length of $c''d''$, measured by the same scale, is 51.4 tons, which is the stress in the diagonal *2-5*.

(*c*) *Stresses in Verticals.* By Art. 22, the stress in the vertical member of a truss with parallel chords is equal to the vertical component of the stress in the diagonal which meets the vertical at a panel-point in the unloaded chord, and is hence equal to the vertical shear in the panel in which that diagonal is located. Thus, the stress in the vertical at panel-point *3* is equal to the vertical shear in the second panel, or 36 tons, etc. The stress in the central vertical at panel-point *7* is equal to the algebraic sum of the shears in the two adjacent panels, or zero in this case.

(*d*) *Character of the Stresses.* The character of the stresses, as to tension or compression, cannot be determined from the equilibrium polygon, but must be ascertained from other considerations. Thus, in any truss the upper chord is in compression and the lower in tension. Again, the positive or negative character of the vertical shear in any panel is known, and, using the method shown in Fig. 25, the nature of the stresses in the diagonals and verticals, respectively in and adjacent to that panel, can be found. In the Pratt truss, the stresses in all verticals are compressive and those in all diagonals in action are tensile; in the Howe truss, the main diagonals have the opposite inclination, and, as the character of the vertical shear is the same, the stresses in the web members are the reverse of those in the Pratt truss. In both trusses, one or more panels at the middle are counterbraced (Art. 19) to provide for the reversal of shear when concentrated loads cross the bridge.

(*e*) *Concentrated Loads.* The graphic analysis of the live load stresses, due to a series of concentrated loads, in this and similar trusses, can be made most conveniently by the use of the force and equilibrium polygons, as in Example 15. The method, while essentially simple, would require more space than is available here, for description in detail. In outline it is as follows :

For convenience in investigation, a tentative truss diagram is drawn with the diagonals all inclined in one way — that of the members, *2–5* and *4–7*, Fig. 32. This uniform inclination does not alter the magnitude of the stress in any diagonal; it simply changes, from tension to compression, the stresses in those members which will have the opposite inclination in the truss as built. Let the series of concentrated loads be those of two coupled locomotives followed by a train, all moving from right to left. Proceeding as in Example 15, the stress curves are drawn and the maximum shear found, both for half-panel intervals from the left abutment to the middle of the truss. From the stress curves, the diagram for maximum bending moments and chord stress can be drawn ; from the maximum shears, the diagram for the latter can be constructed and the stresses in the diagonals determined, as in the shear diagram, Fig. 32. The stresses in the verticals are, in general, equal to the vertical shear in the panel to the right of it.

The stresses, as thus found, are tabulated with those due to dead load, in order to determine the maximum stress in each member and to ascertain which panels require counterbracing, owing to a reversal of stress in the diagonals.

# CHAPTER V

CENTRE OF GRAVITY : MOMENT OF INERTIA

In engineering calculations, the location of the centre of gravity of a plane area is frequently required; for example, through this point the neutral axis of the cross-section of a beam passes. Similarly, the magnitude of the rectangular moment of inertia, about an axis through the centre of gravity of the cross-section, as above, must be known, in order to determine the stresses due to bending at that section of the beam. The centre of gravity and the moment of inertia of plane areas can both be found graphically by the methods given below.

**23. Centre of Gravity.** The weight of a body is the result of the action of terrestrial gravitation upon the latter. A body is composed of an indefinite number of indefinitely small particles; the earth attracts each of the latter with a force proportionate to the mass of the particle. These forces of attraction thus form, for each body, a system whose components are vertical and virtually parallel, since the centre of mass of the earth is 4000 miles distant and each particle is indefinitely near those adjacent to it.

The resultant force of this system is the *weight* of the body, and the line of action of this resultant passes

through the *centre of gravity* or *mass-centre* of the body. In treating areas which have no mass, it is assumed that the force of gravity acting on the area is proportional to the magnitude of the latter, *i.e.*, that there is



Fig. 33

an equal force applied to the centre of each unit of area, as if the latter were that of a thin plate.

**24. Centroid of Two Parallel Forces.** The centre of gravity is virtually the centroid or centre of a system of parallel forces. In Fig. 33, let $P_1$ and $P_2$ be two parallel forces acting from the points of application $A$ and $B$, respectively. Draw the force polygon $Oac$ and

the equilibrium polygon $ADE$; the resultant $R = P_1 + P_2$ acts on the line $GD$ dividing the line $AB$ into segments, $AG$ and $BG$, which are inversely as the forces applied at $A$ and $B$, respectively. That $GD$ is the line of action of $R$ follows from the fact that $R = ac$, and hence the intersection of the sides $AD$ and $ED$, drawn parallel to the rays $Oa$ and $Oc$, must lie at $D$ on a point in the line of action of $R$; this determines that line, since it must also be parallel to the lines of action of $P_1$ and $P_2$.

Now, revolve the forces $P_1$ and $P_2$ about their points of application, forming the system of parallel forces $P_1'$ and $P_2'$, whose force and equilibrium polygons are $O'a'c'$ and $A'D'E'$, respectively. The line of action of the resultant $R'$ of this new system acts on the line $GD'$ which, as before, intersects $AB$ at $G$. With the same relative locations of the poles $O$ and $O'$, the construction of the force polygon $O'a'c'$ is unnecessary, since the sides of the equilibrium polygon $A'D'E'$ are at the same angle with the corresponding sides of the polygon $ADE$ as that through which the forces were revolved.

It is evident that, at whatever angle the forces be inclined, if they remain parallel, the line of action of their resultant will pass through the point $G$, which is hence the point of application of that resultant. This point is called the *centre of parallel forces* or the *centroid*. In Fig. 33, the forces have the same direction. If they act in opposite directions and are still unequal in magnitude, their resultant will be equal to their algebraic

sum and will act on a line parallel to their lines of action and meeting $AB$ prolonged at a point corresponding with $G$ in Fig. 33. If the forces are equal in magnitude and opposite in direction, they form a couple whose centroid is infinitely distant from the points of application of the forces and on a line drawn through these points.

**25. Centroid of Complanar Parallel Forces whose Points of Application are Complanar with all of the Forces, but are not in a Straight Line.** These are the conditions which exist in the graphical determination of the centre of gravity of an area. It is evident that, so long as the points of application of parallel forces are in the same straight line, the centroid of the system will lie at the intersection of the line of action of the resultant with that line, whatever may be the number or relative direction of the forces. When, however, the points of application, although in the same plane, do not lie in the same straight line, the method shown in Fig. 33, *i.e.*, the intersection of the lines of action of resultants, can be used for complanar forces in determining the centroid of the system.

In Fig. 34, let $P_1 \cdots P_4$ be a system of parallel forces having the points of application $A$, $B$, $C$, and $D$, respectively, these points and the lines of action being in the same plane. Draw the force and equilibrium polygons, *Oae* and *EBFLH*, respectively ; the resultant $R$ acts on the line $RHK$. Revolve the forces about their points of application, keeping the lines of action still in their original plane. The system, $P_1' \cdots P_4'$, is thus

I

formed, having the equilibrium polygon $E'B'F'L'H'$.
The line of action of the resultant $R'$ of this system
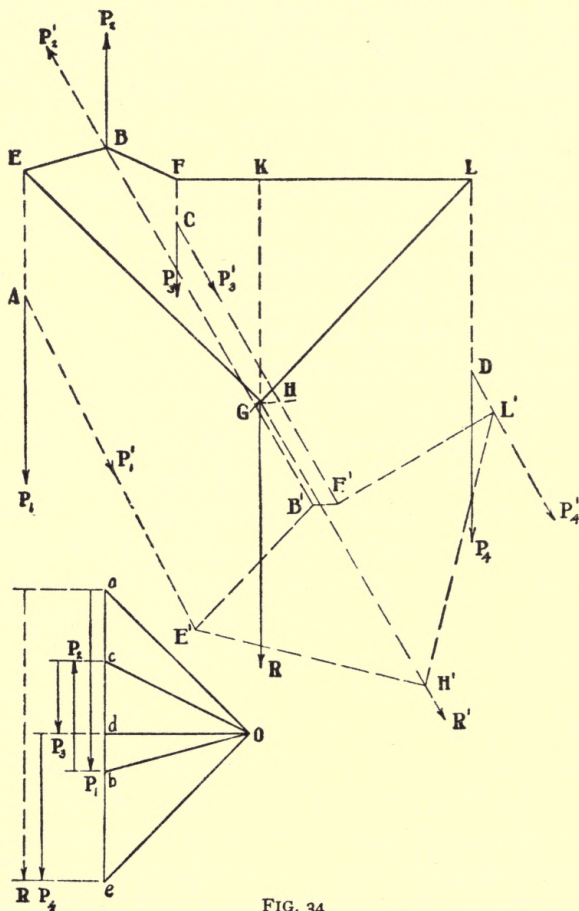


FIG. 34

intersects the line of action of the resultant $R$ at the
point $G$, which point is the centroid of the system.

The intersection of the lines of action of resultants is thus the general method of finding the centre of gravity of an area by the force and equilibrium polygons, the area, if possible, being first divided into segments of geometrical form whose areas and the locations of whose centres of gravity are known.    Then, by applying at each of these centres of gravity a force proportional to its corresponding area, the system of parallel forces, shown in Fig. 34, is established.

If the area whose centre of gravity is required is of irregular shape and incapable of division into segments of geometrical form, its centre of gravity may be located, in close approximation, by drawing a series of vertical lines, closely and uniformly spaced, which divide the area into strips so narrow that the centre line of each space may be regarded as the length of that strip.    At the centre of gravity of each strip, *i.e.*, the middle point of its centre line, apply a vertical force proportional to the area of the strip, that is, to the length of its centre line, since the strips are uniform in width.    Proceeding as in Fig. 34, the centroid of this system of parallel forces, which is the centre of gravity of the area, can be found.

**26. Moment of Inertia : Radius of Gyration.**    (*a*) *Moment of Inertia.*    If a force of magnitude $P$ act at a perpendicular distance $l$ from a given point, its moment about that point is $P \times l$.    As has been stated previously, moments can be treated like forces in graphic processes. Hence, if, on the original line of action, the moment $P \times l$ be assumed to act, its moment about that point will be $Pl \times l = Pl^2$, which is the rectangular moment

of inertia $I$ of the force $P$ about that point.    The
*moment of inertia* is, therefore, the moment of the first
moment, or the *second moment*.

Again, if a particle of mass $m$ be rotating with an
angular velocity $v$, in a plane about a point lying in that
plane and at a distance $r$ from the particle, the angular
momentum of the particle will be $mvr^2$, and that of all
the particles composing the body will be $v\Sigma mr^2$.    The
expression $\Sigma mr^2 = I$ is the sum of the second moments
or moments of inertia of the particles, $m$ being the
quotient of $W$, the weight, divided by $g$, the acceleration
of gravity.    Since, in the mechanics of engineering,
force is taken as the product of mass by acceleration,
this term may be used to describe the second moment
of a force or system of forces about an axis; as an area,
however, is not a material body, the term is applicable
strictly to it only on the assumption that each ele-
ment of area has a mass proportional to its area, as if
the given figure were a thin plate.

In the expression for $I$, as given above, the radius $r$
evidently differs for each particle considered.    Hence,
the *moment of inertia of a body* is the summation of the
products of the masses of the elements of the body by
the squares of their respective distances from the *axis
of inertia* about which the body is assumed to rotate.
Similarly, the *moment of inertia of a force* about such an
axis is the product of the square of the distance between
the point of application of the force and the axis, by the
magnitude of the force; and the moment of inertia of a
system of parallel forces is the sum of these products.

The *moment of inertia of an area* is, in the same way, the summation of the products of each elementary area, considered as a mass, by the square of its distance from the axis of inertia.

(*b*) *Radius of Gyration.*   The radius of gyration is the perpendicular distance from the axis of inertia to the *centre of gyration.*   For a body, the centre of gyration is the point at which, if the entire mass of the body were concentrated in a single particle, the effect of the forces acting on the body would be unchanged and the moment of inertia of the body would remain the same.   The centre of gyration of a body has not an unchangeable location.   Its position may be taken as that of any point in the body which, under the conditions then existing, is at a distance from the axis equal to the radius of gyration ; with any variation in the virtual centre about which the body revolves, that radius changes also and with it the location of the centre of gyration.

If we assume the entire mass of the body to be concentrated at the centre of gyration, at a distance $k =$ the radius of gyration from the axis, then the moment of inertia of the body about that axis is :

$$I = \Sigma m r^2 = k^2 \Sigma m.$$

Hence :

$$k^2 = I / \Sigma m,$$

*i.e.*, the square of the radius of gyration is equal to the moment of inertia of the body divided by its mass.   Similarly, the square of the radius of gyration of a system of parallel forces is equal to the moment of inertia of the system, divided by the magnitude of the resultant $R$ of

the system, or $k^2 = I/R$. For an area $A$, by similar reasoning, $k^2 = I/A$.

**27. Moment of Inertia of a System of Complanar Parallel Forces.** It can be shown, by similar triangles, that the moment of a force about a given point is equal to the product of the pole distance of that force by the intercept which is cut from the line drawn through the given point and parallel to the line of action of the force,
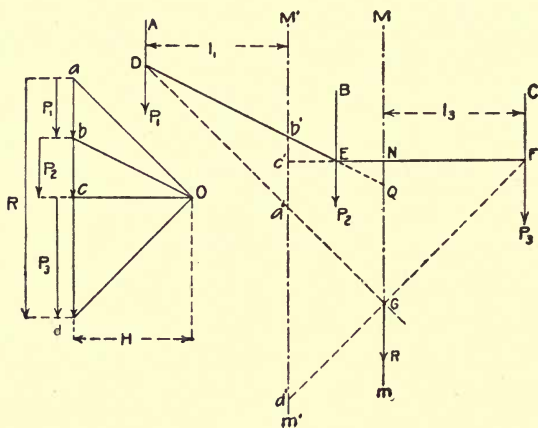


FIG. 35

when the two sides of the equilibrium polygon which intersect on the line of action of the force are prolonged. This principle furnishes one method of finding the moment of inertia of a system of parallel forces. A second, and in some respects more convenient, method, that by the area of the equilibrium polygon, is given below.

In Fig. 35, let $P_1 \cdots P_3$ be a system of complanar parallel forces, acting from the points of application, $A$, $B$, and $C$, respectively. Draw the force polygon $a \cdots d$,

with pole distance $H$, and the corresponding equilibrium polygon $DEFG$. The resultant $R = ad$, and its line of action, the axis $Mm$, passes through the point $G$, and hence through the centroid of the system. For any of the forces, the sides of the polygon which intersect on its line of action and the intercept of these sides on the axis form a triangle, as $FGN$ for the force $P_3$, to which there is a similar triangle, as $Ocd$, in the force polygon. Let $l_3$ be the distance between the line of action of $P_3$ and the axis $Mm$. Then:

$$cd : GN :: H : l_3.$$

Since $cd = P_3$:

$$P_3 l_3 = H \times GN.$$

Multiplying by $l_3$:

$$P_3 l_3^2 = 2\,H(GN \times l_3/2).$$

$P_3 l_3^2$ is the moment of inertia $I$ for the force $P_3$ about the central axis $Mm$, and $GN \times l_3/2$ is the area of the triangle $FGN$. By similar reasoning:

$$2\,H \times \text{area of triangle } DGQ = I \text{ for } P_1,$$
$$2\,H \times \text{area of triangle } ENQ = I \text{ for } P_2.$$

The triangular areas:

$$DGQ + ENQ + FGN = \text{polygonal area } DEFG.$$

Let the area of the equilibrium polygon $= A'$. Then:

$$A' \times 2\,H = I \text{ for } (P_1 + P_2 + P_3) = I \text{ for } R\,;$$

and for the system. If $H$ be made equal to $R/2$:

$$I = A'R,$$

and the square of the radius of gyration is:

$$k^2 = A'.$$

In Fig. 35, the forces $P_1$, $P_2$, and $P_3$ have all the same direction. If one of them, as $P_2$, had a direction

opposite to that of the other two forces, its moment and the corresponding triangular area would have the opposite sign, and the area $A'$ of the equilibrium polygon would then be equal to the algebraic sum of the three triangular areas.

**28. Parallel Axes of Inertia, One passing through the Centroid.** From Art. 27, it will be seen that the moment of inertia of a system of forces about an axis passing through the centroid and parallel to the lines of action of the forces, is the sum of the moments of inertia about that axis of the forces between whose lines of action the axis lies. This principle is general, not only with regard to an area and a body, but also for any given axis parallel to that passing through the centrode. The relation between the moments of inertia about two such parallel axes can be found by either the method of intercepts or that of the area of the equilibrium polygon. The latter method is as follows:

In Fig. 35, let $M'm'$ be any given axis, parallel to the axis $Mm$ through the centroid, and let $l_1$ be the distance of the line of action of the force $P_1$ from the axis $M'm'$. The triangles $Da'b'$ and $Oab$ are similar. Hence:

$$ab : a'b' :: H : l_1,$$
$$P_1 \times l_1 = H \times a'b',$$
$$P_1 l_1^2 = 2\,H(a'b' \times l_1/2)$$
$$= \text{moment of inertia of } P_1 \text{ about } M'm'$$
$$= I' \text{ for force } P_1.$$

Hence, $I'$ for the force $P_1$ is equal to $2\,H \times$ area of triangle $Da'b'$. Similarly:

$I'$ for force $P_2 = 2\,H \times$ triangular area $Eb'c'$,

$I'$ for force $P_3 = 2\,H \times$ triangular area $Fc'd'$.

As before, let the area of the equilibrium polygon $= A'$ and that of the triangle $Ga'd' = A''$.   Then, the moment of inertia of the system is:

$$I' = 2\,H \times \text{polygonal area } DEFGd'a'$$
$$= 2\,H(A' + A'').$$

If $H$ be made equal to one-half the resultant $R$ of the system:

$$I' = R\,(A' + A''),$$

and the square of the radius of gyration is:

$$k^2 = A' + A''.$$

Hence, for an axis passing through the centroid, the moment of inertia is equal to the product of the resultant $R$ by the area of the equilibrium polygon, if the pole distance $H$ be made equal in magnitude to one-half of that resultant.   For a parallel axis, the magnitude of this moment is increased by the product of the resultant by the triangular area formed by the sides intersecting on the line of action of the resultant and the intercept of those sides on the given axis.   For a body of mass $M$ or a cross-section of area $A$, the resultant $R$, as above, is replaced by $M$ or $A$, respectively.

29. **Moment of Inertia of an Area.** — While the general principles established in Arts. 27 and 28 are applicable fully to areas, the determination, with absolute accuracy, of the moment of inertia of a plane area by the methods there given is impossible, since such an area consists of an indefinitely large number of elements of area, and the corresponding system of parallel

forces would therefore be composed of an indefinitely large number of forces, thus making graphic methods unavailable.

(*a*) *Approximate Determination.* — A working approximation, which will serve in most cases in practice, may be made by first determining the centre of gravity of the given area; then dividing the latter into a number of narrow strips parallel to a complanar axis passing through the centre of gravity; and finally applying at the centre of gravity of each strip a force proportional to its area, the whole forming a system of complanar, parallel forces which can be treated by the methods of Art. 27. The greater the number of these strips, and hence the less their width, the nearer the approximation approaches accuracy. With an infinite number of such forces, it is evident that the upper sides *DEF* of the equilibrium polygon, Fig. 35, would be replaced by a curve tangent to the lower sides *DG* and *FG*. In practice, if the given area be divided into a reasonably large number of strips as explained, this curve can be drawn with sufficient accuracy, and the area $A'$ of the equilibrium polygon can then be measured by the planimeter or in other ways.

(*b*) *Accurate Determination.* Both the method of intercepts and that of the area of the equilibrium polygon, as given in Arts. 27 and 28, are applicable in determining, with entire accuracy, the moment of inertia of an area, when the latter can be divided into sections, the area of each of which and its moment of inertia with respect to an axis passing through its centre of gravity

are known. In this case, the force representing the area of the section is applied, not at the centre of gravity of the latter, but at a distance from the given axis of inertia which is equal to the radius of gyration of the area of the section about that axis.

In using this method, a further relation existing between the moments of inertia about parallel axes of inertia may be employed. Thus, let $I$ be the moment of inertia of a system of parallel forces about an axis $Mm$ passing through the centrode of the system, let $M'm'$ be any axis parallel to $Mm$ and at a distance $L$ therefrom, and let $I'$ be the moment of inertia of the system about the new axis of inertia $M'm'$. Then, it can be shown mathematically that:

$$\text{for parallel forces,} \quad I' = I + RL^2,$$
$$\text{for a body,} \quad I' = I + ML^2,$$
$$\text{for an area,} \quad I' = I + AL^2,$$

$R$, $M$, and $A$ being the resultant of the forces, the total mass of the body, and the total area, respectively.

Again, let $a$ be the area of any one of the sections into which the total area is divided, $k$ its radius of gyration about the axis $Mm$ passing through the centre of gravity, $k_1$ its radius of gyration about the given axis of inertia $M'm'$, and $L$ the distance between the two axes. Then (Art. 26):

$$I = ak^2, \ k^2 = I/a.$$
$$I' = I + aL^2 = ak_1^2.$$
$$k_1^2 = Ia^2 + L^2 = k^2 + L^2.$$

The required radius of gyration is therefore the hy-

pothenuse of a right-angled triangle whose sides are $k$ and $L$, the magnitudes of which are known. The force corresponding with the sectional area $a$ is then assumed to act at a distance $k_1$ from the axis of inertia and to be parallel to that axis. If the total area can thus be divided into geometrical figures to which this principle can be applied, either the method of intercepts or that of the area of the equilibrium polygon can be used, with entire accuracy, for the determination of the moment of inertia of a plane area.

## EXAMPLES

18. CENTROID OF PARALLEL FORCES DUE TO LOCO-MOTIVE WHEEL LOADS. Let it be required to find the centroid of the forces due to the locomotive wheel loads in Example 15, viz. 5.75 tons on the pilot wheel and 10.94 tons on each of the four drivers. The load-spacing, from left to right, is: 100, 66, 66, and 72 inches.

If it be assumed that these loads are held in equilibrium by an upward force $P$ equal to their sum, we shall have a system in equilibrium which can be treated by the general method. In Fig. 36 lay off the five loads on the span segment $AA'$ and on the load line $ab$; then $ba = P$. Draw the force polygon $Oab$ and the corresponding equilibrium polygon $BCDEFG$. The vertical line $Mm$, passing through the point $G$, is then the line of action of the force $P$ and also of the resultant $R$ of the forces, which resultant is equal and opposite to $P$ and acts through the centroid of the system. This centroid is located at a point on the line $Mm$ prolonged,

whose location depends on the diameters of the pilot and driving wheels.   Thus, the point of application of the resultant of the forces on the four driving wheels lies on the line of centres of these wheels. If a line be drawn from this point to the centre of the pilot wheel, the intersection of this line with the line *Mm* prolonged will be the centroid of the system, or the point of application of the resultant *R*. The centroid thus lies about 10 inches to the right of the centre of the second driving wheel.
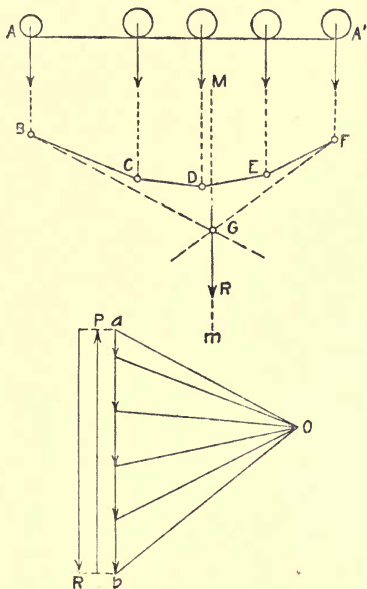


FIG. 36

19. CENTRE OF GRAVITY OF BULB ANGLE.  When a cross-section has an axis of symmetry, its centre of gravity will lie on that axis, and but one equilibrium polygon need be constructed.  The bulb angle shown in Fig. 37 is not thus symmetrical, and hence equilibrium polygons must be drawn for two directions of the forces.

The dimensions of the angle are : total depth, 6 inches; width of flange, 3 inches; thickness of web, $\frac{1}{2}$ inch; of flange, tapering from $\frac{1}{2}$ inch to $\frac{7}{16}$ inch; curves of bulb, $\frac{1}{2}$ inch radius; of flange, $\frac{5}{16}$ inch radius.

For graphical analysis, the cross-section is divided into sections, whose areas in square inches are: *abc*, 0.12; *bcde*, 1.02; *dhkl*, 2.75; *efg*, 0.02; *klo*, 0.2; *pqr*, 0.05; *prso*, 0.5; *rst*, 0.4. The total area of the cross-section is thus 5.06 square inches.

At the centre of gravity of each of these sections
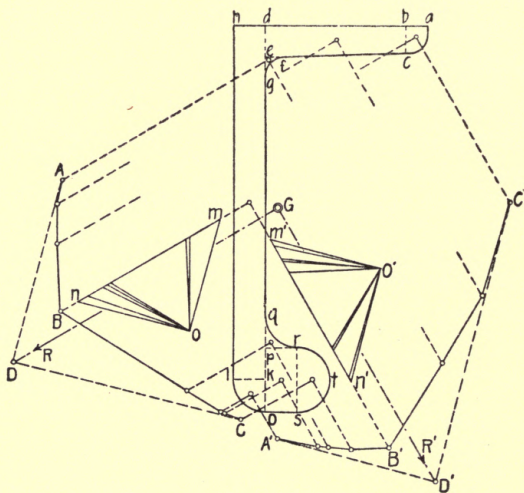


FIG. 37

there is assumed to be applied a force equal to the area of that section. The centre of gravity of a rectangle lies at the intersection of the diagonals; of a triangle, on a line drawn from any vertex to the middle of the opposite side, and at a distance from the vertex equal to two-thirds the length of this line; of a circular sector, on its middle radius at a distance from the centre of the circle equal to $2\,RC/3\,A$, in which $R$ is the radius, $C$ is

the length of the chord of the arc, and $A$ is the length of the arc. For a 90-degree sector, $A = 2\pi R/4$. The centre of gravity of a trapezoid can be found by dividing it into two triangles and connecting their centres of gravity. The latter, for the trapezoid, will lie on this connecting line, dividing it into segments inversely proportional to the areas of the two triangles so that the algebraic sum of their moments about it shall be zero.

In Fig. 37, the forces are first assumed to act downward and to the left. The centre of gravity of each section is located and a force equal to the area of the section is laid off on the load line $mn$. Assuming equilibrium as in Example 18, the force and equilibrium polygons $Omn$ and $ABCD$ are drawn. The resultant $R$ of the forces acts on the line $DG$. The forces are now revolved through 90 degrees, and the force and equilibrium polygons $O'm'n'$ and $A'B'C'D'$ are similarly constructed. The line of action of the new resultant $R'$ is $GD'$. The two lines of action meet at $G$, which is the centre of gravity of the bulb angle. The point $G$ is vertically distant 2.86 inches from the face of the flange and 3.14 inches from the base of the bulb.

20. CENTRE OF GRAVITY OF A PARTIAL AREA. The area of a perforated plate is its total area, less the combined areas of the perforations. In determining the centre of gravity of such a plate, the area of each perforation must evidently be treated as an upward, opposing force, in order to consider the deductions as above.

Thus, Fig. 38, let it be required to determine the

centre of gravity of the partial area, *ABCDEF*, which area is equal to that of the rectangle *ADEF*, less those
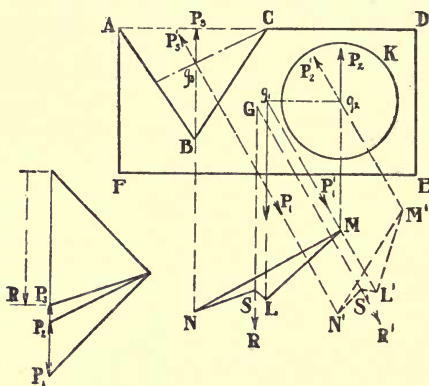


FIG. 38

of the triangle *ABC* and the circle $g_2K$. At the centres of gravity $g_1$, $g_2$, $g_3$ of these three figures, apply parallel forces $P_1$, $P_2$, and $P_3$ respectively proportional to the areas, the forces corresponding with the two deducted areas being assumed to act in the opposite direction from $P_1$, which represents the area of the rectangle. Draw the force polygon and the equilibrium polygons, *LMNS* and *L'M'N'S'*, for two directions of the system of forces. The resultants *R* and *R'* intersect at the point *G*, which is the centroid of the system and the centre of gravity of the partial area, *ABCDEF*.

21. MOMENT OF INERTIA OF THE CROSS-SECTION OF A DECK BEAM: APPROXIMATE METHOD. — Figure 39 represents the cross-section of a deck beam. It is required to determine the moment of inertia of this cross-section, about an axis passing through the centre of gravity of the cross-section and perpendicular to the web, using the approximate method of Art. 29.

The dimensions are: depth of beam, 6 inches; width

of flange, $4\frac{3}{8}$ inches; thickness of web, $\frac{9}{32}$ inches; of flange, $\frac{17}{32}$ tapering to $\frac{7}{32}$ inches; depth of bulb, $1\frac{1}{16}$ inches; thickness, $1\frac{11}{32}$ inches; area of cross-section, 4.1 square inches.

For the reasons stated in Art. 29, the cross-section is divided into a number of relatively narrow vertical



Fig. 39

strips, and the area and centre of gravity of each strip are found. It is assumed that, at each of these centres of gravity, there is applied a force equal to the area of that section, the whole constituting a system of verti-cal parallel forces, whose resultant $R = A = 4.1$ square inches, the area of the cross-section. Assuming equilib-

rium (Example 18), the force and equilibrium polygons *Omn* and *BCDE* are drawn. Since the cross-section is symmetrical about its central axis, the centre of gravity lies on this axis, the line of action of the resultant $R$ is $GE$, parallel to those of the forces, and $G$ is the centrode of the system of forces and the center of gravity of the cross-section.

By Art. 27, the moment of inertia of the cross-section about the axis $GE$ is $I = RA' = AA'$, in which $A'$ is the area of the equilibrium polygon. It is evident that, with the construction shown in Fig. 39, the area $A'$, as measured from the drawing, must be an approximation only to the true area. This approximation becomes closer as the width of the strips is decreased, until, when that width is indefinitely small, the broken line $BCD$ becomes a curve which is tangent to the lines $EB$ and $ED$, prolonged, at the points $b$ and $d$, vertically below the ends of the cross-section. With the approximate construction as in Fig. 39, the measured area is thus greater than the true area, since the sides of the polygon form chords of the true curve. In this case, a full-scale drawing, made with fair accuracy, gave an area, measured by planimeter, of $A' = 5.34$ square inches, from which $I = 4.1 \times 5.34 = 21.89$, as against the value 21.6 stated in the manufacturer's handbook and obtained from an area bounded by the true curve.

**22. ACCURATE DETERMINATION OF THE MOMENT OF INERTIA OF AN AREA ABOUT AN AXIS PASSING THROUGH ITS CENTRE OF GRAVITY.** — Let it be required to determine accurately, by the methods of Art. 29, the

moment of inertia of the geometrical figure shown in Fig. 40, about an axis passing through its centre of gravity. This fig-
ure is composed of two rectangles and a triangle, whose areas, beginning at the left, are: $a_1$ = 1.25, $a_2$ = 1.5, $a_3$ = 0.56 square inches. The total area is therefore $A$ = 3.31 square inches. By constructing the force polygon $Opq$, with pole distance $A/2$, and the corres-ponding equilib-



FIG. 40

rium polygon (not shown), the centre of gravity $G$ of the figure is located.

For an accurate determination, the force represent-ing an area should be applied (Art. 29) at a distance from the given axis of inertia equal to the radius of gyration of this force about that axis. Further, it has been shown (Art. 29) that, if $k_1$ be the radius of gyra-tion of an area about an axis passing through its centre of gravity, $K_1$ the similar radius about a parallel axis, and $l_1$ the distance between the two axes, then $K_1^2 = k_1^2 + l_1^2$.

In this case, the given axis of inertia is $M'm'$ passing through the point $G$. Applying the principles as above to the area $a_1$, draw, from any point $c$ in the axis $M'm'$, a horizontal line intersecting at $b$ the line of action $gb$ which passes through the centre of gravity of $a_1$, and, from $b$, erect $bb' = k_1$, the radius of gyration of $a_1$ about the axis $gb$. Then, $cb = l_1$ and $cb' = \sqrt{k_1{}^2 + l_1{}^2} = K_1$, the radius of gyration of the area $a_1$ about the axis $M'm'$. Revolving $cb'$ to $cb''$, we have the new line of action $b''B$ of the force corresponding with the area $a_1$, which line of action lies to the left of the original line.

In the same way, the new lines of action $dD$ and $eE$, for the forces corresponding with the areas $a_2$ and $a_3$, respectively, are determined. The values of $k_1$, $k_2$, and $k_3$ are found analytically. Thus, the radius of gyration of a rectangle about an axis passing through its centre of gravity and perpendicular to the base, is equal to the length of the base, divided by the square root of $12$; the similar radius for a triangle about an axis parallel to the base, is equal to the altitude, divided by $\sqrt{18}$. Hence, $k_1 = 0.14$, $k_2 = 0.87$, and $k_3 = 0.18$. For clearness, $k_1$ and $k_3$ are exaggerated in Figure 40.

While the forces have thus new lines of action, they have still the same magnitudes, and hence the original force polygon $Opq$ will serve. Constructing the corresponding equilibrium polygon $BDEF$, the line of action of the resultant $R = A$ is found to be $Mm$ passing through the point $F$. The problem is thus reduced to the case analyzed in Art. 28 and Fig. 35, viz. there are

two parallel axes of inertia $Mm$ and $M'm'$, the former passing through the centrode of the system; the moment of inertia of this system about the axis $M'm'$ is therefore $I' = A(A' + A'')$, or the product of the area of the cross-section by the area of the polygon $BDEF$, plus that of the triangle $FJL$, formed by the intersection of the sides $BF$ and $EF$ and the intercept cut by those sides from the axis $M'm'$. Substituting the value of the area $A$ and those of the two latter areas, as measured from the diagram, $I' = 6.55$.

# CHAPTER VI

The resistance due to friction is both helpful and hurtful in the performance of mechanical work. It is friction which gives the driving wheels of a locomotive the grip on the rails which enables them to move the train; and, again, it is the friction of the car wheels on the track which, disregarding the resistance of the air, forms the total work of pulling the train and develops the total stress in the draw-bar. The analysis of the action of friction is, in some cases, complex, but it is often simplified materially by the use of graphic methods.

**30. Friction.** The sliding friction of solids — which only will be treated herein — is the resistance to relative motion of surfaces in contact and under pressure. This resistance is caused by the interlocking of the minute projections and indentations of these surfaces. If the latter were absolutely smooth and perfectly hard, there would be no projections to disengage and override, no frictional resistance would occur, and hence no mechanical work would be required to produce relative motion. In practice, the action of pure friction, as above, is complicated by adhesion, abrasion, the viscosity of lubricants, etc.

*Sliding friction* is the friction of plane surfaces; the friction of warped surfaces, such as screw threads, and of cylindrical surfaces, such as journals, is a modified form of this action. *Rolling friction* is the friction of a curved body, as a cylinder or sphere, when moving over a plane surface or one of greater curvature. In this case, contact occurs theoretically on a line or point only; but, as all materials are more or less elastic, there is actually a surface of contact, and therefore rolling friction is identical in cause with sliding friction. The resistance or 'force' of friction acts in the plane of contact of plane surfaces, and, with curved surfaces, along their common tangent. In most mechanisms, the *friction of motion* only is encountered; with belting, the *friction of rest* occurs. The latter is greater theoretically than the former since, during rest, the harder body has better opportunity to indent and engage the softer surface with which it is in contact. The slightest jar, however, nullifies this action.

The force of sliding friction is:

$$F = fN,$$

in which $F$ is the total resistance or force which opposes the relative motion of two surfaces in contact, $N$ is the total pressure normal to those surfaces, and $f$ is the *factor* or *coefficient* of *friction*. This formula, which expresses the far from well-established laws of sliding friction — assumes that the total force of friction is independent of the area of the surfaces in contact, of their relative velocity, and of the intensity of pressure,

*i.e.*, the pressure per unit of surface. According to this expression, $F$ is equal simply to the product of the total normal pressure by the factor *f*. If the normal pressure and velocity be low, and the surfaces dry or but slightly lubricated, *f* may be considered as having a constant value for the same materials and state of surfaces; but, in the wide range of conditions met in practice, it has been found that the value of *f* is affected also by the velocity, the intensity of pressure, the temperature of the surfaces, and the viscosity of the lubricant, so that the formula, as above, is incomplete and approximate. If, however, these effects be all considered in the value assigned to the factor *f*, and this value be determined independently for each case, the formula will hold.

From the formula for $F$, we have:

$$f = F/N = \tan \phi,$$

*i.e.*, the coefficient of friction is the tangent of an angle known as the *angle of friction* $\phi$. Thus, in Fig. 41, let $AB$ and $AC$ represent planes hinged at $A$ and support-ing a body $D$ of weight $W$. Keeping $AC$ horizontal,
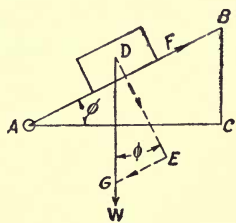


FIG. 41

let $B$ be raised until the limiting condition for equilibrium of the body $D$ is attained, that is, until with any further elevation of $B$ the body will slide downward. Let the angle $BAC = \phi$. Resolve $W$ into components $DE$ and $EG$, perpendicular and parallel, respec-tively, to the surface $AB$. The force of friction $F$ is

produced by the former of these components which is the total normal pressure $N$ on the contact-surfaces, and $EG$ is equal in magnitude and opposite in direction to $F$. Hence:

$$F = fW \cos \phi = W \sin \phi, \text{ and } f = \tan \phi.$$

**31. Friction of Plane Surfaces: Friction Cone.** Let Fig. 42 represent a body of weight $W$ resting on a horizontal plane $AB$. While no force acts but that of gravity, the body will be held in equilibrium by the weight $W$, acting from the centre of gravity $O$, and by the reaction $R = N$ of the plane $AB$. This reaction is equal to and opposed to $W$, has the



FIG. 42

same line of action, is normal to the contact-surface, and is the resultant of the infinite number of infinitely small reactions from that surface.
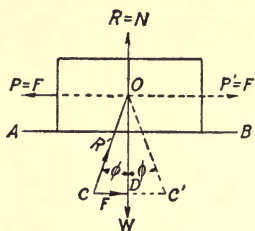
Now assume a force $P = F$ as applied. Let the magnitude of this force be such that, with any increase, it will cause motion to the left, the limiting condition of equilibrium having been reached; it is therefore opposed by the full force of friction $F$. The body is now in equilibrium under the action of the forces, $W$, $R$, $P$, and $F$. The resultant of $R$ and $F$ is the *virtual reaction* $R'$ or $CO$, making the angle of friction, $\phi$, with the line of action of $R$. Since equilibrium prevails, $W$, $R'$, and $P$ meet at a common point $O$. For motion to the right, similar but reversed conditions exist, as is shown by

dotted lines. In either case, the supporting plane $AB$, the normal reaction $R$, and the force of friction $F$ may be replaced by the virtual reaction $R'$, making the angle of friction with the normal to the contact-surface, and in such a direction that its component parallel to that surface will oppose the relative motion of the two bodies.

It will be seen that the limiting condition for equilibrium, with regard to motion in any direction, is that the



virtual reaction shall lie on the surface of the cone described by the revolution of the triangle $OCD$ about the normal $OD$. This is the *friction cone* or *cone of resistance*. When the force $P$ is less than $F$, $R'$ falls within the surface of the cone and there will be no motion; when $P$ is greater than $F$, $R'$ lies beyond the cone, and motion ensues.

When the contact-surfaces are inclined, the conditions are the same as with level surfaces, except that, owing to the inclination, the weight $W$
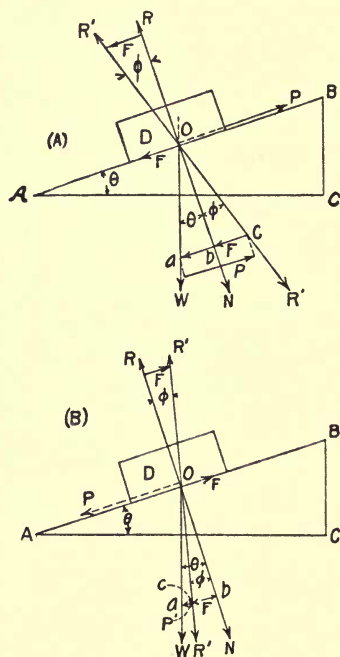
has a component acting along the incline and tending to pull the body downward. Thus, let Fig. 43 represent

an inclined plane of angle $BAC = \theta$, on which a body of weight $W$ rests. In the upper diagram $(A)$, let $P$ be a force acting along $AB$ and just sufficient to move the body upward on the incline. Momentary equilibrium exists under the action of the weight $W$, the normal reaction $R$ which is equal to and opposed to the normal pressure $N$, and the force of friction $F$ which acts downward along $AB$. The resultant of $R$ and $F$ is the virtual reaction $R'$, inclined to $R$ by the angle of friction. Resolving $W$ and $R'$, parallel and perpendicular to $AB$, we have:

$$P = ac = ab + bc = W \sin \theta + F.$$

In diagram $(B)$, let $P'$ be the force which is just sufficient to move the body downward on $AB$. The force $F$ now acts upward. Resolving as before:

$$P' = ac = ab - bc = W \sin \theta - F.$$

**32. Friction of Screw Threads.** The screw thread is essentially but an inclined plane wrapped around a cylinder. In a square thread, the radial elements of the thread surface are perpendicular to the axis of the cylinder; in a triangular thread, these elements are inclined to that axis. The axial load $W$ is borne usually by the bolt, whose thread thus corresponds with the contact-surface of the body $D$ in Fig. 43; the nut thread is then similar in its action to the surface $AB$ in that figure, the nut being supported by a bearing surface.

The pressure on these threads is assumed to be concentrated on the mean helix, or the circumference of

the mean thread-diameter $d$, of pitch angle $\alpha$, as in Fig. 44. Each element of the thread surface is re-garded as sustaining an equal elementary portion of the total load or stress $W$ on the bolt, and each element has therefore a frictional resistance of the same magni-



FIG. 44

tude. Since the conditions for all elements are thus identical, the total thread resistance, the axial load, and the external turning forces on the nut may be assumed to be each equally divided and concentrated at two points, 180 degrees apart, on the circumference of diameter $d$. The forces $P$, for lifting the load, thus

form a couple whose arm is $d$; and similarly the forces $P'$, for lowering, have the same arm and points of application.     In Fig. 44, these points are $H$ and $K$.

(*a*) *Square Threads*.     In Fig. 44, taking the nut as the turning member, let $ABC$ be the inclined plane formed by developing one convolution of the nut thread of mean diameter $d$; $AB$ is the contact-surface of that thread, and $EG$ represents a portion of a bolt thread. The base of the plane is $\pi d$, its height is the pitch $p$, and the pitch angle is $BAC$.     Consider the forces $P$ or $P'$ as applied to the nut in a plane normal to the axis, and as tangent to the mean thread circumference.

When the nut thread is on the point of moving to the left to raise the load, its front half, whose center of pressure is at $H$, is in equilibrium under the action of one of the forces $P$, the reaction $R'$ making the friction angle with the normal reaction $R$, and the reaction $R_1 = W/2$ of the nut support, which reaction is vertical as the friction of the support is neglected.    In the force polygon, lay off $ab = R_1$, and draw $bc$, $bc'$, and $ca$ parallel, respectively, to the lines of action of $R'$, $R$, and $P$.

Then,          $P = ca = W \tan(\phi + \alpha)$.

When the nut thread is about to lower the load under the action of the force $P'$, the virtual reaction at the contact-surfaces becomes $R_2'$, making the friction angle with the normal reaction.    Equilibrium then exists under the action of the force $P'$ and the reactions $R_1$ and $R_2'$. Drawing $bd$ parallel to the latter,

$$P' = da = W \tan(\phi - \alpha).$$

In order to eliminate all friction but that of the screw thread, the section $EG$ of the bolt thread has been assumed to have no lateral motion.

(*b*) *Triangular Threads*.   In Fig. 44 *a*, let $N$ and $N'$ be the normal pressures on square and triangular threads, respectively.   Then, $N' = N \sec \beta$, in which $\beta$ is the base-angle of the triangular thread.   If $F$ and $F'$ be the forces of friction for the two threads, we have, since $F = fN$:

$$F' = fN' = fN \sec \beta = F \sec \beta.$$

Hence, as compared with the square thread of the same pitch angle, the friction $F'$ of the triangular thread is $\sec \beta$ times greater.   To determine the force $P$ in Fig. 44 for these conditions, prolong the lines of action $R$ and $R'$, and, at any point, draw the line $ef$ perpendicular to $R$.   This line is proportional to the force $F$. From $e$ draw $eg$ making the angle $\beta$ with $ef$; from $f$ drop the perpendicular $fg$ on $eg$, thus determining the line $eg$ which is proportional to the force $F'$.   Revolve $eg$ to $eh$ on $ef$ prolonged, and draw $Hh$ which is then the line of action of the virtual reaction $R''$ of the triangular thread.   In the force polygon, lay off $bc''$ parallel to the line of action of $R''$, and the force $P$ will then be $ac''$.

**33. Pivot and Collar Friction**.   When the lower end of a vertical shaft, subjected to end thrust, is supported and guided by a step bearing, the end of the shaft forms a pivot journal; the latter may be plane, conical, globular, etc.

(*a*) *Plane Pivots*.   This form is shown in Fig. 45.

To analyze graphically, let the total load on the shaft be $W$, and, as with the screw thread, assume it to be equally divided and one-half concentrated at each of the two points $A$ and $A'$, diametrically apart, on the circle of radius $r$ described on the disk of radius $r_1$. The total normal pressure on the bearing
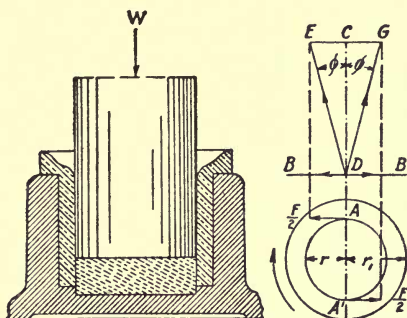


FIG. 45

is $W$; the intensity of pressure, or average pressure per square inch, is $W / \pi r_1^2$. The total force of friction is $F = fW$, and, assuming uniform pressure over the disk, the frictional resistance per unit of area is $fW / \pi r_1^2$.

To determine the relation between the radii $r_1$ and $r$, consider the circumference $2\pi r$ to be an elementary ring of width $dr$ and of area $2\pi r dr$. The total normal load on this ring is:

$$w = 2\pi r dr \times W / \pi r_1^2.$$

Multiplying $w$ by the coefficient of friction $f$ gives the frictional resistance of this elementary ring, and this product, multiplied by the radius $r$, is the moment of this elementary resistance, or:

$$wfr = m = \frac{2fW}{r_1^2} \cdot r^2 dr.$$

The integral of this expression, between the limits $r_1$

and zero, gives the moment of the total frictional load or force $F$, which is:

$$M = \frac{2\,fW}{r_1^{\,2}} \int_0^{r_1} r^2 dr = \frac{2}{3}\,fWr_1.$$

Dividing this moment by its force, $F = fW$, we have the mean radius at which $F$ acts, or:

$$M / F = \tfrac{2}{3}\,r_1 = r.$$

In the diagram, $BB$ is the plane of the disk, $CD$ is one-half $W$, and $DE$ and $DG$ are the virtual reactions at the points $A$ and $A'$, respectively.

(b) *Conical Pivots.* Fig. 46 represents a cone-shaped pivot. Let $\theta$ be the half-angle of the cone, $r_1$ the radius of the upper end of the journal, and $W$ the total axial load. As with the plane pivot, consider one-half of $W$ as concentrated at each of two points, 180 degrees apart, on an elementary strip of the conical surface of radius $r$ and width $dr$. Resolve $W$ in the directions of the two normals $NN$ to the contact-surface at these points. Then, $N = W / 2 \sin \theta$, and the total force of friction is:
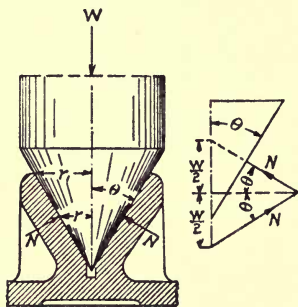


FIG. 46

$$F = 2\,fN = fW / \sin \theta.$$

By using the same method as with the plane pivot, the mean radius at which the total force of friction acts will be found to be $\tfrac{2}{3}\,r_1$. Hence, a diagram similar to

that in Fig. 45 will show the forces graphically, due regard being had to the change in the value of $F$.

(*c*) *Mean Radius of Friction.* For the two cases discussed, this radius is theoretically $\frac{2}{3} r_1$, where $r_1$ is the greatest radius of the bearing. This is true of all pivot bearings whose projected area is circular, and not annular. This theoretical value assumes a uniform intensity of pressure and a constant value of $f$ for the whole surface of the bearing. These conditions do not exist in practice, and a mean radius of friction equal to one-half the greatest radius of the bearing surface of the pivot is generally taken as more correct. With this change, the formulæ, as deduced above, hold.
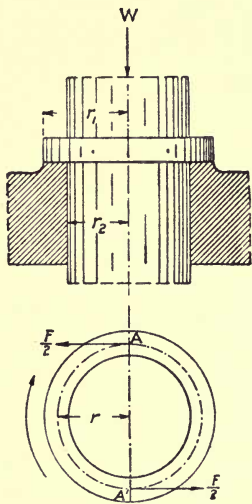


FIG. 47

(*d*) *Collar Friction.* As shown in Fig. 47, the collar-bearing is simply the plane pivot reduced to annular form. Let $W$ be the total axial load on the shaft, and assume it to be equally divided and one-half concentrated at each of the points $A$ and $A'$. The external radius of the bearing is $r_1$; its internal radius is $r_2$; and the mean radius of friction is $r$. The value of $r$, in terms of $r_1$ and $r_2$, is to be found.

The total force of friction is $fW$, and, assuming a uniform pressure over the whole surface of the bearing, the frictional resistance per unit of area is $W/\pi(r_1^2 - r_2^2)$.

The area of an elementary ring of radius $r$ is $2\pi r dr$. The total load on this ring is:

$$w = 2\pi r dr \times W/\pi(r_1^2 - r_2^2).$$

The frictional resistance of this load is $fw$ and the moment of this resistance is:

$$m = fw \times r = fr \times 2\pi r dr \times W/\pi(r_1^2 - r_2^2).$$

The integral of this expression between the limits $r_1$ and $r_2$ gives the moment of the total frictional load or force $F$, which moment is:

$$M = \frac{2fW}{\pi(r_1^2 - r_2^2)} \int_{r_2}^{r_1} r^2 dr = \frac{2}{3} \cdot \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \cdot fW.$$

Dividing this moment by the force $F = fW$, the mean radius of friction is:

$$r = \frac{2}{3} \cdot \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}.$$

**34. Journal Friction; Friction Circle.** — (a) *Journal friction* is complex in analysis, since the fit is, in most cases, more or less free, and pure sliding friction between cylindrical surfaces rarely occurs in ordinary machinery, although in the mechanism of precision it is the rule. The usual shaft journal rests in its bearing somewhat loosely, giving theoretically a line bearing between cylindrical surfaces of different radii, which is rolling friction. When the shaft revolves, friction causes the journal to roll up the side of the bearing, like a car-wheel moving up an inclined track. The journal ascends until the weight makes it slide backward, and this rise and fall continue with every variation in the coefficient of friction. The method of analysis which follows is that first established by

Rankine and later developed by Hermann and others.
It applies to close-fitting bearings only.

In Fig. 48, let $A$ be a cylindrical journal at rest in the
bearing $B$; the
forces acting are
the weight or
other vertical pres-
sure $W = CD$ and
an equal and op-
posite reaction
from the bearing
at the point $G$.
Now, let there be
applied a turning
force $P$, normal to
a radius and of
such magnitude
that the limiting
condition of equi-
librium is reached,
the frictional re-
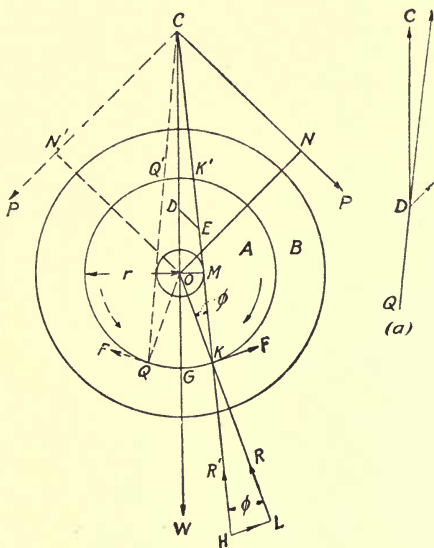


FIG. 48

sistance is overcome, and the journal is on the point of
beginning clockwise rotation. The forces acting are
then $W$, $P$, and the virtual reaction $R'$ from the bearing,
whose line of action is yet to be determined.

Prolong the lines of action of $W$ and $P$ until they
meet at $C$; lay off $CD = W$ and $DE$ equal to $P$ and
parallel to the latter's line of action. Then, $CE$, the
resultant of $W$ and $P$, is equal in magnitude, opposed in
direction, and has the same line of action as the virtual

reaction $R'$ from the bearing, since $W$, $P$, and $R'$ are in equilibrium and their lines of action must meet at a common point. Draw $CEH$, and, from the centre of pressure $K$, lay off $KH = R'$. This reaction is simply the resultant of the indefinite number of indefinitely small reactions from the bearing.

Draw the line of action $OL$ of the normal reaction $R$ through the point $K$; resolve $R'$ normally and tangentially. Then, $R = LK$ is the total normal pressure on the journal, and the force of friction is $F = HL = KL \times$ coefficient of friction $f$. Since the virtual and normal reactions are inclined from each other by the angle of friction, the angle $HKL = OKE = \phi$.

(b) *Friction Circle*. In Fig. 48, drop the perpendicular $OM$ from the centre $O$ on the line of action of the reaction $R'$. Then, if $r$ be the radius of the journal $OM = r \sin \phi$. If now the radial arm $ON$ and the force $P$ be revolved through all positions about the centre $O$, it will be found that, while the location of the point of intersection $C$ will be changed, the perpendicular distance of the line of action of $R'$ from the centre will be equal in every case to $r \sin \phi$. Hence, the locus of such points of intersection as $M$ is a circle described from $O$ and of radius $r \sin \phi$. This is the *friction circle*.

The principle is general, applying to all forms of loading of a closely fitting journal and bearing, either of which has motion in any direction relatively to the other. The dotted lines show the effect of anti-clockwise rotation, the force $P$ having the new line of action $CN'$, but the other conditions remaining the same. If

it be assumed that the journal is fixed and the bearing revolves — like a connecting rod pulling a crank pin — the magnitude and lines of action of $R'$ will be the same for rotation in either direction, but the centres of pressure will be changed from $K$ and $Q$ to $K'$ and $Q'$, respectively. Again, assume that the pressure acts upward instead of downward, as at $(a)$, Fig. 48, and that the rotation is clockwise, the force $P$ acting from $N'$ to $C$. If the bearing be the moving member, the centre of pressure will be at $Q'$, at which point $R'$ will act downward on the journal; if the journal be the moving member, the load will act upward and $R'$ downward at the point $Q$. It will be seen that the friction circle and a tangent to it, which is the line of action of the virtual reaction $R'$, wholly replace either the journal or the bearing in the graphical investigation of their friction. The general laws of sliding friction may be applied to determine on which of these tangents $R'$ acts in any given case.

**35. Link Connections: Friction Axis.** A *link* is a straight, rigid machine member employed to transmit power between two rotating or oscillating members, or one rotating and one sliding member, as is the case with the connecting rod. Journal friction therefore exists at each end of the link; the principles governing its action are those established in Art. 34. If there were no friction, the radius of the friction circle would be zero and the force would be transmitted from one member to the other along the line joining the centres of the journals or bearings of the link. With friction,

the resultant of the transmitted force and the force of friction acts along the *friction axis*, which is a line tangent to the friction circles of the two journals.
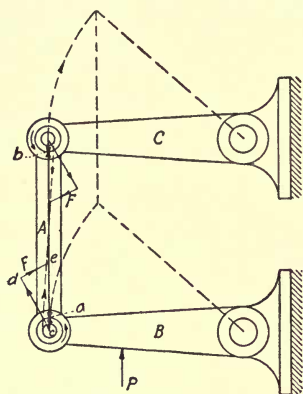


FIG. 49

There are four fundamental cases of this action, two of which are represented in Figs. 49 and 50 and the others deduced therefrom. In each of these mechanisms, $A$ is a link and $B$ and $C$ are hinged levers, $P$ is a force acting upward on the lower or driving lever, and the dotted lines show the path in ascending.

It will be observed that:

(*a*) The link may be assumed to be in momentary equilibrium in any given position, but its efficiency, as thus determined, applies to that position only. Also, the direction of the friction axis depends on which of the connected levers is the driver, and
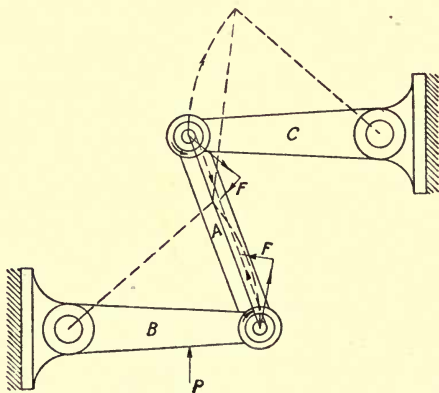


Fig. 50

hence on whether the link is in tension or compression.

(*b*) A force and its reaction may be regarded as two equal and opposite forces having the same line of action.   Since the link is in equilibrium, the friction axis is the line of action of a force acting from one journal and an opposing reaction from the other.

(*c*) The link bearings move on the journals which they support, the journals being thus relatively stationary.   The direction of this motion is determined by that of the mechanism.

(*d*) Since the link is in equilibrium, the force of friction at either journal must so act as to oppose the relative motion of the bearing.   This consideration determines the direction of the friction axis.   If the direction of rotation is the same with both bearings, the axis will lie diagonally, being tangent to one friction circle on one side, and to the other on the opposite side. If the bearings rotate in opposite directions, the friction axis will be parallel to the geometric axis.

In Fig. 49, the lower lever is the driver.   Hence, disregarding friction, the force on the lower journal is concentrated at the point *a* on the geometric axis, acting there on the lower bearing of the link.   This puts the latter in compression, and its force acts on the lower side of the upper journal at the corresponding point *b*.   From the path of the mechanism, the motion of both bearings is seen to be anti-clockwise.   At the lower bearing, the tangential force of friction, $F = de$, acts from left to right, and therefore on the left side

of the friction circle, where it and the normal pressure
line *cd* combine to form the resultant pressure line *ce*.
Hence, the friction axis is here tangent to the left side
of the friction circle.

At the upper journal, $F$ must act from right to left,
combining with the reaction from the journal to form
the virtual reaction which, acting along the line *ec*,
must be tangent to the right side of the friction circle.
If, on the other hand, the upper lever be the driver in
lifting the connected parts, the direction of motion of
the bearings will be the same, but the link will be in
tension and the friction axis will be tangent to the other
side of each friction circle.

In Fig. 50, the lower lever is the driver and the link
is again in compression, but the direction of rotation of
the two bearings is not the same.    Hence, the forces of
friction act on the same sides of the two friction circles
and the friction axis is parallel to the geometric axis.
If, again, the upper lever be the driver in lifting the
connected parts, the direction of motion of the bearings
will not be changed, but the link will be in tension, and
the friction axis, while still parallel to the geometric
axis, will be tangent to the other sides of the friction
circles.

**36. Chain Friction : Resistance of Ropes to Bending.**
(*a*) *Chains*.   The friction of a chain in passing over a
chain drum or sprocket wheel, is a modified form of
journal friction, the pins joining the links constituting
the journals.   Let Fig. 51 represent a chain pulley, of
effective radius $R$, revolving in a clockwise direction.

The load on the advancing or left side of the chain is
$W$; the driving force on the right or receding side is $P$.

Let $r$ be the radius
of the pins joining
the links, and $r_1$ that
of the wheel journal.

Relative motion
of the links of the
chain occurs only
when the latter
bends at the joints
on reaching and
leaving the horizon-
tal diameter $EF$ of
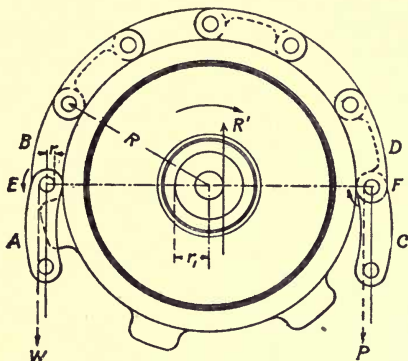


FIG. 51

the wheel. At this time, the advancing link $A$ turns in
an anti-clockwise direction on the link $B$, the latter serv-
ing as a bearing; and, similarly, the link $C$ is rotated in
the same direction with regard to the link $D$. From
Art. 34, it will be seen that the effect of these actions is
to remove the lines of action of the load $W$ and the
force $P$ from the vertical lines passing through the
centres of the link-pins over to the left by the distance
$r \sin \phi$, i.e., the friction of the joints of the chain in-
creases the leverage of the load and decreases that of
the driving force by that amount. Similarly, the line
of action of $R'$, the vertical reaction of the wheel-bear-
ing, is removed to the right through the distance $r \sin \phi$.
$R'$ is the resultant of $P$ and $W$. Hence:

$$P = W \cdot \frac{R + (r + r_1) \sin \phi}{R - (r + r_1) \sin \phi}.$$

(*b*) *Ropes*. The resistance of a rope in passing on or off a sheave or grooved pulley has an effect similar to that of chain friction on the lines of action of the load and driving force, *i.e.*, the lever arm of the load is increased, and that of the driving force decreased, by the same amount in both cases. This effect is not due, however, to the same causes as with the chain. In the rope, it is produced, when the latter is bent, by the relative motion and consequent friction of the strands, by the compression of the fibres on the inner side and the stretching of those on the outer in winding on, and by the reversal of this action when the rope leaves the pulley.

Thus, if we assume the rope to be wound on a pulley, like the chain in Fig. 51, it will be bent in advancing when it passes at $E$ above the horizontal diameter $EF$, and will be straightened again when it descends below $F$ at the right. In the first of these operations, the normal forces acting on the horizontal cross-section of the rope at $E$ are the tension at the centre due to the load, the tension on the outer half from the stretching of those fibres, and the opposing compressive force on the inner half. The resultant of these three forces is a tension which acts at a distance, which for convenience we shall call $s$, outward from the centre of the rope, so that, neglecting the friction of the pulley journal, the lever arm of the weight $W$ is $R + s$, $R$ being the distance from the centre of the pulley to that of the rope.

When the rope straightens at $F$, the same forces act, but those of tension and compression from bending change places, occurring, respectively on the inner and

outer halves of the cross-section.   Hence, the resultant
tensile force acts between the pulley- and rope-centres,
at the distance $s$ from the latter, so that, neglecting
journal friction, the lever arm of the driving force $P$ is
$R - s$.   It will be seen that the distance $s$ thus corre-
sponds with the radius of the friction circle for chain
friction; and, as with the latter and considering journal
friction, we may write :

$$P = W \cdot \frac{R + r_1 \sin \phi + s}{R - r_1 \sin \phi - s}$$

in which $r_1 \sin \phi$ is the radius of the friction circle for
the journal of the pulley.

The value of $s$ can be expressed only by empirical
formulæ, the constants of which have been derived
from experiment.   This follows, since the resistance of
a rope to bending varies directly as the tensile stress in
it due to the load or the driving force; directly as some
power of its diameter, since the smaller the rope, the
greater its pliability; and inversely as the radius of the
pulley, since the greater this radius, the less the re-
quired bending.   Evidently, it also depends upon the
material of the rope, its length of service, etc.

**37. Belt Gearing.**   The friction of a belt on a pulley
in power transmission is the friction of rest and not
that of motion; it is not the cause of lost work, but the
means by which useful work is done in preventing rela-
tive motion of the working parts; and the greater
within practical limits it becomes, the better.    With
regard to the friction of motion of belt gearing — that
of the pulley shaft in its bearing — the reverse is true,

its efficiency being much less than that of toothed gear-
ing, since the power transmitted is directly proportional
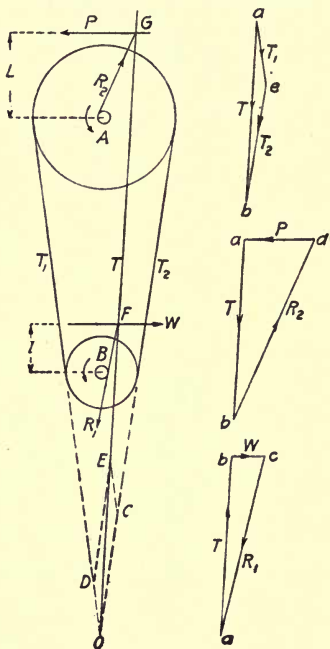


FIG. 52

to the difference between
the tensions of the tight
and slack sides of the belt,
while the aggregate thrust
on the bearing which pro-
duces journal friction is
due to the pull of both of
these tensions.

The graphics of belt
gearing present no prob-
lems which have not been
examined previously here-
in. Figure 52 represents
a pulley $A$ driving through
belting a pulley $B$, the ro-
tation being anti-clockwise.
Each pulley may be con-
sidered as in equilibrium
under the action of a driv-
ing force or a resistance,

the two belt tensions, the normal reaction of the bearing,
and the journal friction, the latter being relatively large.
The angle of contact $\theta$ between belt and pulley can be
found by construction, and the ratio between the two ten-
sions can be determined from the well-known formula:

$$\frac{T_2}{T_1} = E^{f\theta},$$

in which the angle $\theta$ is given in circular measure, $E$ is

the base of the Napierian system of logarithms, $T_2$ and $T_1$ are, respectively, the tensions of the driving and slack sides of the belt during motion, and $f$ is the coefficient of friction.

Prolong the lines of action of these tensions until they intersect at $O$; and, from $O$, lay off $OC$ and $OD$ of such dimensions that $OC/OD$ is equal to the tension ratio just found. Their resultant $OE$ gives the line of action of the resultant $T$ of $T_2$ and $T_1$, which line intersects the lines of action of the resistance $W$, of lever-arm $l$, and the driving force $P$, of arm $L$, at the points $F$ and $G$, respectively. From the latter points and tangent to the friction circles, draw the lines of action of $R_1$ and $R_2$, the virtual reactions of the bearings.

The driven pulley $B$ is in equilibrium under the action of the resistance $W$, the resultant $T$ of the tensions, and the virtual reaction $R_1$, the magnitude of $W$ and the lines of action of all the forces being known. In the lowest force triangle, lay off $bc = W$ and draw $ba$ and $ca$ parallel, respectively, to the lines of action of $T$ and $R_1$, thus determining the magnitudes of the two latter. To find the magnitude of $P$, consider the driving pulley as similarly in equilibrium and lay off, in the second triangle, $ab = T$ and draw $ad$ and $ab$ parallel, respectively, to the lines of action of $P$ and $R_2$. The magnitude of the tensions is determined in the upper force triangle by resolving the resultant $T$ of the tensions parallel to the lines of action of its components, $T_1$ and $T_2$. The effect of journal friction can be shown by replacing $R_1$ and $R_2$ by reactions passing through the

centres of the respective journals, and drawing the corresponding sides of the force triangles parallel thereto, which method will give the values, excluding journal friction, of $P$, $T$, $T_1$, and $T_2$ for the same resistance $W$.

**38. Friction of Gear Teeth.** — In the transmission of power by gear wheels, a part of the lost work is expended in overcoming the frictional resistance of the teeth. This is due to the fact that a pair of teeth, while engaged, move one upon the other, the line of bearing changing continually on both contact surfaces and the relative motion of the latter being a combination of rolling and sliding.



FIG. 53

Figure 53 (*a*) represents the positions, at the beginning and end of engagement, of the same pair of involute teeth on the spur gears $A$ and $B$, the wheel $A$ being the driver and rotating in a clockwise direction. The two pitch circles are tangent at the point $p$ on the line joining the centres of the wheels. The arcs $bp$ and $cp$ are the *arcs of approach*; $pb'$ and $pc'$ are the *arcs of recess*. The line $aa'$ is the *line of action*, which is the

path of the points of contact of the teeth during engagement. This line is normal to the contact-surfaces and in properly formed teeth it always passes through the point $p$, making an angle with the horizontal called the *angle of obliquity of action.*

In *approach*, the points $b$ and $c$ gradually draw nearer until they meet at $p$. Since the arc $ab$ is shorter than the arc $ac$, it follows that the flank of the driving tooth rolls through a distance equal to $ab$ on the face of the driven tooth, and slides for a distance equal to $ac - ab$. In *recess*, this process is reversed, the face of the driver rolling on the flank of the driven tooth through a distance equal to $c'a'$, and sliding through the distance $b'a' - c'a'$.

Figure 53 (*b*) shows the engaged portions of the two gear wheels $A$ and $B$. In this figure, it is assumed that the lengths of the arcs of action, $bpb'$ and $cpc'$, in (*a*), are such that two pairs of teeth are simultaneously engaged, and that the normal pressure is the same between the teeth of each pair. The line of action is $aa'$ as before.

If there were no friction, the reaction due to the load on the driven teeth *3* and *4* would act on the drivers *1* and *2* along the line $a'a$ from right to left, and this reaction would be exactly equal to the force $P$, exerted by the driving wheel and acting in the direction $aa'$ and on that line. From the enlarged force polygon (*c*), it will be seen that, considering friction, the reaction from tooth *3* takes the direction $da$, and that from tooth *4*, the direction $a'd$, both reactions being inclined to the normal by the angle of friction $\phi$ and intersecting the line of centres at the point $d$. If these virtual reactions

be resolved vertically and parallel to $aa'$, the vertical components will neutralize each other and the true line of action of the force and the resultant reactions will become $a_1'a_1$, parallel to $a'a$ and at a perpendicular distance therefrom of $\tan \phi \times aa'/2$, which, assuming $aa'$ to be equal to the circular pitch, becomes $\tan \phi \times$ pitch/2.

The friction of spur gear teeth is then, in its effect, similar to chain friction. With frictionless motion, both the force and resistance would act along the normal $aa'$; with friction, the line of action is shifted to the parallel line $a_1a'_1$. This change increases the distance from the centre of gear $A$, at which the load from gear $B$ acts on $A$, and decreases the distance from the centre of gear $B$, at which the driving force from gear $A$ acts on $B$. Hence, in general, the leverage of the power is lessened and that of the load is increased by the distance determined above, $\tan \phi \times$ pitch/2.

Figure 53 and this discussion refer to involute teeth. With cycloidal teeth, the line of action is not a straight line, as $aa'$, but is an arc of the circle with which the profiles of the teeth are described. The loss from tooth friction and the wear in service are, in consequence, less than with any other form of tooth.

### EXAMPLES

23. FRICTION OF STATIONARY ENGINE. The readiest practical approximation to the power expended in overcoming the friction of an engine can be obtained by taking indicator cards with the engine unloaded and running at various speeds. There are two important objections to this method : the errors of the indicator

and the fact that the results found include all resistances, so that it is impossible to separate that for any particular bearing. On the other hand, the difficulty with analytical or graphic methods is that the coefficient of friction cannot be estimated with much accuracy, since
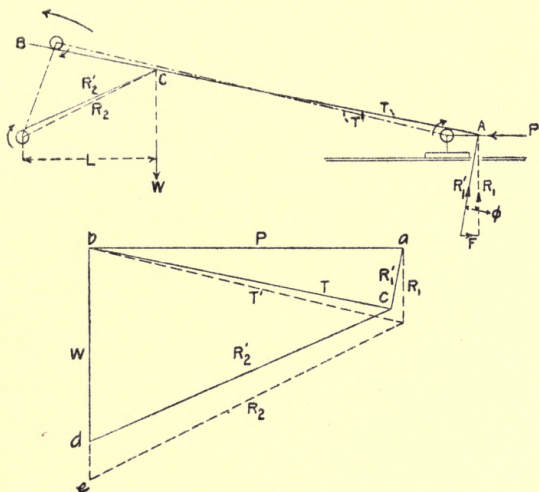


FIG. 54

it appears to vary not only with pressure, but with velocity, the temperature and condition of the contact-surfaces, the viscosity of the lubricant, etc. The graphic method for the crosshead, crank pin, and shaft bearings is shown in Fig. 54.

The data are: cylinder diameter, 15 inches; stroke, 18 inches; length of connecting rod, 36 inches; maximum unbalanced pressure on piston, 100 pounds per square inch; corresponding total force acting on piston before cutoff, 17,671 pounds.

M

In the position shown in Fig. 54, the crosshead is approaching the mid-point of the forward stroke, and hence the angle with the horizontal made by the connecting rod is increasing, while that between the connecting rod and crank is decreasing. Only centre lines and the friction circles are shown. In computing the radius of these circles, the same diameter, 6 inches, will be taken for the crosshead, crank pin, and shaft, since the friction circle is so small that even a considerable variation in these diameters will affect but little its radius, in the graphic method illustrated here. The crosshead slipper reciprocates and the bearing of the crosshead pin oscillates, while the crank pin and shaft are rotating journals. The coefficient of friction will hence vary considerably for each. For convenience, an average coefficient, $f = 0.185$, will be taken for all, with a consequent angle of friction $\phi$ of 10° 30′. Then, the radius $r$ of each journal is $6/2 = 3$ inches, and the radius of the friction circle is $r \sin \phi = 3 \times 0.182 = 0.546$ inch.

From the principles given in Art. 35, the small arrows, showing the direction of the frictional resistance, are described at each bearing, and the friction axis $AB$ is drawn, as the line of action of the thrust $T$ on the connecting rod. The crosshead pin is in equilibrium under the action of $T$, the piston-force $P$, and the virtual reaction $R_1'$, inclined to the left by the angle $\phi$ from the normal $R_1$ to the slipper bearing. These three forces must meet at the common point $A$, which is the intersection of the lines of action of $P$ and $T$; this determines the location of $R_1'$. In the force polygon, lay off $ab = P$

and draw $ac$ and $bc$, parallel respectively to the lines of action of $R'_1$ and $T$.   Then, $bc = T$.

The direction of the line of action of the resistance of the load to the motion of the shaft depends upon what the latter drives and upon where the power is transmitted from this driven member.   For compactness in the drawing, assume in this case that there is a gear wheel, 2 feet in diameter, fixed to the shaft and engaging a similar wheel which is set on the engine centre-line and between the shaft and crosshead.   It is required to determine, for the shaft-driven gear, the magnitude of the resistance $W$, of arm $L = 1$ foot, which, at this position of the engine, the force $P$ will overcome.

The crank, the shaft, and the gear wheel fixed to the latter form virtually a bell-crank whose horizontal arm is of length $L$.   This bell-crank is acted on by the forces $T$ and $W$, the normal reaction $R_2$ from the bearing, and the frictional resistance to the shaft's motion, which resistance acts in a clockwise direction.   The lines of action of $T$ and $W$ meet at the point $C$ and the virtual reaction $R_2'$ from the shaft bearing acts, by Art. 35, on a line drawn from $C$ and tangent to the upper side of the friction circle.   In the force polygon, the intersection of $cd$ and $bd$, drawn parallel to the lines of action of $R'_2$ and $W$, respectively, give $W = bd = $ 11,000 pounds.

Disregarding friction, the forces are $T'$, $R_1$, and $R_2$, as is shown by dotted lines.   Constructing the force polygon for these conditions, we have $be = W = $ 13,200

pounds. For this position of the engine, its efficiency is therefore $11/13.2 = 83.3$ per cent, and the power



FIG. 55

expended in overcoming the friction of these four bearings is 16.7 per cent of the total force $P$ on the piston.
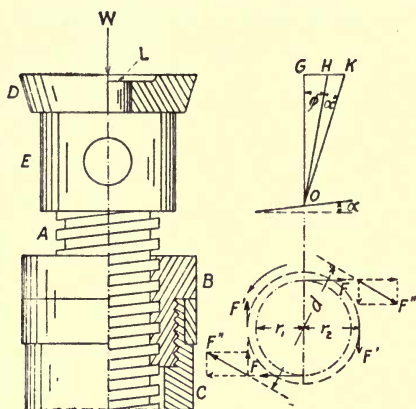
24. FRICTION OF SCREW JACK. Figure 55 represents the upper part of an ordinary screw jack. The screw $A$ engages the nut $B$, fixed in the supporting frame $C$; the load $W$ rests on the swivel plate $D$, journalled on the screw head $E$, the latter being bored for the bar by which power is applied to turn the screw and lift the load.

The data are: nominal or outside diameter of screw, $2\frac{9}{16}$ inches; depth of thread, $\frac{1}{4}$ inch; diameter at base of thread, $2\frac{1}{16}$ inches; pitch of screw, $\frac{1}{2}$ inch; pitch angle $\alpha$ of mean helix (Art. 32), $4°$, about.

Carefully conducted laboratory experiments on a well oiled screw jack gave, for the resistance of the thread, a coefficient of friction $f = 0.1494 = \tan \phi$, and hence an angle of friction $\phi$ of $8°30'$. These results are doubtless somewhat lower than those usual in practice.

Let the circle of radius $r_1 = 1\frac{5}{32}$ inches be the horizontal projection of the mean helix and assume the load

$W$ to be equally divided and its halves concentrated at the upper and lower extremities of the vertical diameter of this circle. If $F$ be the horizontal component of the thread resistance on one-half of the mean helix and $P$ be the total horizontal force acting, equally divided as a couple, at the ends of the diameter $2\,r_1$ on the bar to raise the load,
then one-half or $P/2$ will be taken as acting at the same points as $F$.



FIG. 56

The screw rotates in an anticlockwise direction to raise the load, the bolt thread thus moving upward on the stationary nut thread. The conditions are then as shown in Fig. 56, in which $BAC$ is the pitch angle, $AB$ is the upper surface of the nut thread, and $D$ is a portion of the bolt thread representing one-half of a convolution. This section $D$ is in momentary equilibrium under the action of the load $W/2$, the virtual reaction $R'$ making the angle of friction $\phi$ with the normal reaction $R$, and the horizontal raising force $P/2$. Hence:
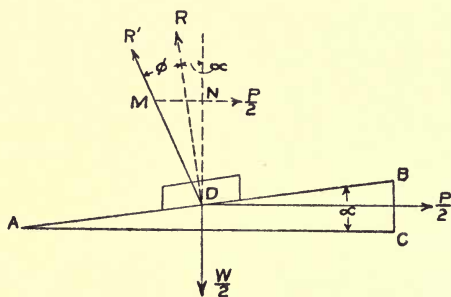
$$MN = \frac{P}{2} = \frac{W}{2}\tan{(\alpha + \phi)},$$

which is also the magnitude of the force $F = GK$ (Fig. 55), acting at each of the two points of application of the load $W$ on the mean helix.

The tangent of $\alpha + \phi$ is 0.222. Taking $W$ as 10 tons (20,000 pounds) $P/2 = 2220$ pounds, which is one of the forces of the horizontal couple acting at the extremities of the diameter $2\,r_1 = 2\frac{5}{16}$ inches to raise the load; it is also equal to the force $F$ of the resisting couple acting with the same arm.

This calculation assumes that the load rotates with the jack screw, which is seldom the case. If the load be stationary, the friction of the swivel plate $D$ must be considered. The mean radius of friction (Art. 33) of this plate is (Fig. 50) $r_2 = 1\frac{1}{4}$ inches. Taking the same angle of friction, and assuming that one-half of the load $W$ is concentrated at each of two points diametrically apart, as before, the force of friction $F'$ ($GH$ in Fig. 50) acting at the extremities of the diameter $2\,r_2$ is 10,000 × 0.1494 = 1494 pounds.

There are thus two couples which, for simplicity, may be considered as acting in the same plane: $P/2 = F = 2220$ pounds with arm of $2\frac{5}{16}$ inches, and $F' = 1494$ pounds with arm of $2\frac{1}{2}$ inches. These two couples may be assumed to act at any points in their respective circumferential paths. Taking the points of application as 90 degrees apart, as in Fig. 55, the resultant couple is found to have a force $F'' = 2666$ pounds with an arm $d$ of 3.32 inches. If the raising bar be of such length that its couple, as actually applied, has an arm of 24 inches, the force required at each of its ends will be about 368 pounds.

Since it is not possible to make the load $W$ absolutely central, there will also be journal friction on the pin $L$

on which the swivel-plate is centred. The amount of this resistance will depend on the eccentricity of the load and the resulting normal pressure on the pin. If this pressure be known, the frictional resistance can be determined by the methods of Art 34.

25. PULLEY BLOCKS: RELATION OF LOAD AND POWER. Professor Hermann[1] gives the method shown in Fig. 57 for finding the relation of load and power, and the ten-sions in the vari-ous portions of a chain or rope pass-ing over the sheaves of the pair of blocks of a tackle, friction be-ing considered.

In Fig. 57, the blocks are three-sheaved; the load $W$ is suspended from the lower or movable block $A$, and the frame $C$ of the upper or fixed
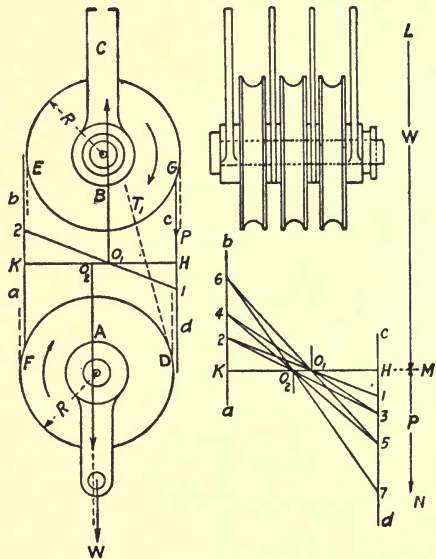


FIG. 57

block $B$ is supported from above. The rotation of the sheaves in raising the weight is clockwise, the rope winding on at $D$ and $E$ and off at $F$ and $G$. If the

[1] "Graphical Statics of Mechanism," Hermann-Smith, New York, 1904, p. 88.

upper or fixed end be secured to the hanger $C$, as at $T_1$, there will then be seven portions of the rope whose tensions, $T_1$ to $T_7$, are to be determined — the last, or that of the hauling end, being equal to the lifting force $P$.

For any two consecutive sections of these seven — one on each side of a sheave of either block — the tension in the rope on one side constitutes the load, and that on the other, the power for the pair. If $R$ be the effective radius of the sheave, $r_1$ that of the block-journal, and $t$ be equal to $r \sin \phi$ or to $s$, as in Art. 36, then the relation between the power $p$ and the load $w$ on these two portions will be, for either a chain or a rope:

$$p = w \cdot \frac{R + r_1 \sin \phi + t}{R - r_1 \sin \phi - t}. \tag{1}$$

At the journal of the lower block, the load $W$ acts on the vertical tangent to the left (Art. 35) of the friction circle; at the upper journal, the reaction of the support $C$ acts on the similar tangent to the right. Again, from the figure, it will be seen that the unwinding or power side is the left on the lower block and the right on the upper, the tensile forces in the sections of chain or rope acting, in each case, at a distance $R - t$ from the centre, as shown by the full lines $ab$ and $cd$; similarly, on the respectively opposite, or load, sides of the blocks, the tensile forces act at a distance $R + t$ from the centre. Hence, as the lower block is free, it will, when ascending, swing to the left for a distance $2\,t$, as shown in the figure, so that the tensile forces shall act

vertically. When upward motion ceases, the block swings backward until it is again vertically below the upper block ; when the load is lowered, the process is reversed.

In passing from the fixed end of the chain or rope to the free or hauling end, its tension is increased, whenever it passes over a sheave, in the ratio given by the reciprocal of the fraction in the preceding equation. Hence, starting at the fixed end, the general expression for the ratio between the tensions in two consecutive sections is :

$$\frac{T_n}{T_{n+1}} = \frac{R - r_1 \sin \phi - t}{R + r_1 \sin \phi + t}. \qquad (2)$$

In the left-hand figure, draw the horizontal line $HK$, cutting the lines of action of the rope-tensions $ab$ and $cd$ at $K$ and $H$, respectively, and those of the load $W$ and the reaction from the upper bearing at $o_2$ and $o_1$, respectively. Then, $HK = 2R$ and the distances :

$$Ho_1 = Ko_2 = R - r_1 \sin \phi - t,$$
$$o_1 o_2 = 2 r_1 \sin \phi + 2 t,$$
$$Ho_2 = Ho_1 + o\,o_2 = Ko_1 = R + r_1 \sin \phi + t.$$

Assume that the tension $T_1$ in the first section from the fixed end is known and is equal to $H$–$1$ on the line $cd$. From $1$, draw through $o_1$ the line $1$–$2$ cutting the line $ab$ at $2$. Then, $K$–$2$ is equal to the tension $T_2$ in the next succeeding section, for the triangles $H$, $1$, $o_1$ and $K$, $2$, $o_1$ are similar, and :

$$\frac{H, 1}{K, 2} = \frac{Ho_1}{Ko_1} = \frac{R - r_1 \sin \phi - t}{R + r_1 \sin \phi + t} = \frac{T_1}{T_2}. \qquad (3)$$

Continuing in the right-hand diagram, draw from the point $2$ through $o_2$ the line $1$–$2$ and by similar reasoning $H$–$3$ is the tension $T_3$ in the third section. In the same way, $K$–$4 = T_4$, $H$–$5 = T_5$, $K$–$6 = T_6$, and $H$–$7 = T_7 = P$ are found. The sum of the tensions, $T_1$ to $T_6$, inclusive, which is the line $LM$, is equal to the load $W$, while the force $P$ is, on the same scale, equal to $MN$.

Since the distances $Ho_1$, $Ho_2$, $Ko_1$, and $Ko_2$ are constant for any system of sheaves such as is shown in the figure, it is evident that the ratio $MN/LM$ is constant, and that any value of $T_1$ may be assumed in finding the ratio. When the latter is determined, the value of $P$ for any given value of $W$ is given by the expression:

$$P = W \cdot MN/LM.$$

As an example, consider a $3\frac{3}{4}$-inch hemp rope ($1\frac{1}{4}$ inches diameter), passing over a pair of 3-sheaved blocks, as in Fig. 57. Take the power $P$, applied at the hauling end, as 1000 pounds, or slightly above 800 pounds per square inch of sectional area of the rope; and let the effective radius $R$ of the pulley, *i.e.*, that to the centre of the rope, be 6 inches, the diameter of the pin or shaft of each block be $1\frac{1}{4}$ inches making $r_1 = \frac{5}{8}$ inch, and the coefficient $f$ of journal friction be 0.1, giving $\phi = 6°$, about. It is required to determine the weight $W$ which the given power $P$ will lift.

It is necessary first to find the value of $t$, in order to draw the lines of action $ab$ and $cd$. This value is also that of $s$ in Art. 36. It cannot be calculated with any great degree of accuracy for reasons given previously.

Various empirical expressions have been used.  Eytelwein's formula, as employed by Reuleaux and Weisbach,[1] gives, for the total stiffness or resistance to bending $S$ of a hemp rope in both winding on, and unwinding from, a pulley:

$$S = 0.472 \, Wd^2/R,$$

in which $W$ is the load and $d$ and $R$ are respectively the diameter of the rope and the radius of the pulley, both in inches.

Assuming that the resistance to winding is equal to that of unwinding — which is only approximately true — we have, for either operation:

$$S/2 = 0.236 \, Wd^2/R.$$

Now, if the rope were absolutely elastic, $S$ would not exist, and the moment of the weight would be $W \times R$. Considering stiffness, this moment is, by Art. 36, $W(R + s)$.  The added moment $Ws$ is therefore equivalent to that produced by $S/2$ acting at the radius $R$. Hence:                 $Ws = SR/2.$

Substituting :

$$s = 0.236 \, d^2 \text{ inch.}$$

While the results given by this formula are somewhat high, it is sufficiently accurate for large hemp rope under heavy stress.  For rope of other materials, the constants require modification.  Substituting the value of $d$, $1\frac{1}{4}$ inches :

$$s = t = 0.368 \text{ inch.}$$

---

[1] " Mechanics of Engineering and Machinery," New York, 1896, Vol. I, § 197.

With $\phi = 6°$, the radius of the friction circle:

$r_1 \sin \phi = 0.625 \times 0.1045 = 0.065$ inch.

In Fig. 57, draw the upper sheave, the friction circle and the dotted tangent at $G$. Let fall the tangent at $B$ until it meets at $o_1$ the indefinite horizontal line $HK$. Draw the line of action $cd$ parallel, to the left, and at a distance $t$ from the tangent at $G$; this line meets $HK$ at $H$, thus locating the latter point. From $o_1$ set off to the left the distance $o_1 o_2 = 2(r_1 \sin \phi + t) = 0.866$ inch; from $o_2$, drop the tangent $A$ to the lower friction circle, and describe the latter to its right, thus determining the centre of the lower sheave. Erect the dotted tangent at the left of this sheave, and, at a distance $t$ to the right of it, draw the line of action $ab$, intersecting $HK$ at $K$.

Assume any value — say 500 pounds — for $H$–$1$ and lay it off on the line $Hd$. Then, as explained previously, draw $1$–$2$, $2$–$3$, $3$–$4$, etc. Adding $H$–$1$, $K$–$2$, $H$–$3$, $K$–$4$, $H$–$5$, and $K$–$6$, and dividing their sum by $H$–$7$, we have 3.7, which is the value of the ratio $W/P$. Hence $W = 3700$ pounds. If stiffness and friction were not considered, the value of this ratio would be 6. The efficiency of the tackle is thus 61.66 per cent, which is a fairly accurate result. This method assumes that the section $T_1$ of the rope is vertical and thus parallel to the other sections.

The value of the ratio in Equation (3) is 0.865. This value is constant throughout the system, and gives the relation between the tensions in any two consecutive sections of the rope.

## 26. Spur Gears: Relation of Load and Power.

Figure 58 represents the pitch circles of a train of spur gears, $A$, $B$, and $C$, with involute teeth. The power applied to the driving gear $A$ is equivalent to a force $P$ of arm $L$; the resistance acting on the gear $C$ is equal to a force $W$ of arm $L'$. Assuming friction at the wheel journals and between the engaged teeth, it is required to determine the magnitude of the force $P$ for a known resistance $W$.

The driving gear $A$ is in equilibrium under the action of the force $P$, the reaction $T_1$ from the teeth of gear $B$, and the virtual reaction $R_1$ from the bearing. By Art. 38, the reaction $T_1$ is parallel to the line $aa'$ passing through the point of tangency of the pitch circles, the distance of $T_1$ from the centre of $A$ being greater than that of $aa'$ by the amount given in the article cited. The



FIG. 58.

lines of action of $P$ and $T_1$ intersect at $c$, and the reaction $R_1$ drawn from $c$ is, by Arts. 34 and 35, tangent to the lower side of the friction circle.

The intermediate gear $B$ is in equilibrium under the
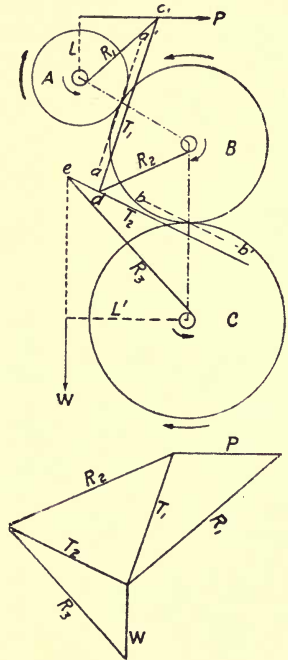
action of the force $T_1$, the reaction $T_2$ from the teeth of gear $C$, and the virtual reaction $R_2$ from the bearing. Since the gear $B$ is the driver for gear $C$ and its motion is opposite to that of gear $A$, the reaction $T_2$ has, by Art. 38, the direction and location shown in the figure, being parallel to the line $bb'$ passing through the point of tangency of the pitch circles. The lines of action of $T_1$ and $T_2$ intersect at $d$, from which point the reaction $R_2$ is drawn tangent to the lower side of the friction circle.

The driven gear $C$ is in equilibrium under the action of the resistance $W$, the force $T_2$, and the reaction $R_3$ from the bearing. The lines of action of $T_2$ and $W$ meet at $e$, from which point the reaction $R_3$ is drawn tangent to the upper side of the friction circle.

The directions of all the forces acting on the train and the magnitude of one, the resistance $W$, are known. Starting with $W$, the force triangle for each gear and the force polygon for the train can be drawn by the general method, as shown by the diagram below.

For these gears, the angle of obliquity (Art. 38) is usually about 15 degrees, that is, the line $bb'$ makes an angle of 15 degrees with the horizontal.

# PROBLEMS

**1.** What weight can be drawn up an inclined plane rising 1 in 5 by a pull of 200 pounds: (*a*) when the pull is parallel with the plane; (*b*) when it is horizontal? Disregard friction.

**2.** A wheel of 5 feet diameter weighs 2 tons, including its load. What is the least horizontal force necessary to pull it over a stone 4 inches high?

**3.** A boiler weighing 5000 pounds is supported by tackles from the fore and main yards. If the tackles make angles of 30 and 35 degrees, respectively, with the vertical, what is the tension in each of the two?

**4.** Sheer legs 50 feet long are spread 18 feet at the base. The back stay is 80 feet long. Find the stresses acting in each member when lifting a load of 25 tons at a distance of 20 feet from the foot of the sheer legs, the weight of the latter being disregarded.

**5.** A crane post is 12 feet high; its jib, 32 feet long, and stay, 23 feet long, meet at the peak *A*. Two backstays, making angles of 45 degrees with the horizontal, are in planes due north and due west from the post. A weight of 6 tons is suspended from *A*. Find the forces in the jib and stays: (*a*) when *A* is southeast of the post; (*b*) when *A* is due east; (*c*) when *A* is due south.

**6.** A rod weighing 10 pounds rests in a smooth hemispherical bowl which is fixed with its rim horizontal. The rod is 15 feet long, and a length of 3 feet is outside of the bowl. If the inclination of the rod to the horizontal be 30 degrees, find the reactions of the bowl.

**7.** A simple triangular truss, 24 feet span and 3 feet deep, is supported at the ends and carries a load of 3 tons concentrated at the middle. Find the stresses in each member.

**8.** The span of a roof is 16 feet; length of rafters, 9 and 12 feet, respectively. The rafters are spaced 2 feet apart and the roofing material weighs 16 pounds per square foot. Find the thrust on each rafter and the stress in the tie bar.

**9.** Let a roof truss, whose general plan is that shown in Fig. 11, be unsymmetrical, the rafters to left and right having inclinations of 50 and 45 degrees, respectively, to the horizontal. The span is 45 feet; the trusses are spaced 10 feet apart; and the weight of the roof covering and snow is 40 pounds per square foot of roof surface. Find the stresses in the members.

**10.** Find the stresses in the members of this truss due to wind load, assuming the right end of the truss to be free. Take the horizontal wind pressure as 40 pounds per square foot.

**11.** A king post truss has a span of 16 feet and a rise of 8 feet. Find the stresses in the members due to a load of 18,000 pounds at the middle.

**12.** A floor beam, 18 feet long and carrying a uniform load of 180 pounds per linear foot, is trussed by rods which are $1\frac{1}{2}$ feet below the middle of the beam. Consider the rods as jointed in the middle of the beam, and find the stress in each rod. Disregard the weight of the beam.

**13.** A simple beam, 18 feet long, carries a load of four tons. Draw the bending moment and shearing force diagrams: (*a*) when the load acts at the middle of the beam; (*b*) when it acts 5 feet from one of the ends. Disregard the weight of the beam.

**14.** A simple beam is 18 feet long and loaded with one ton evenly distributed along its span. Find the bending moment and the shearing force at a distance of 6 feet from one end and also at a distance of 1 foot from the middle. Disregard the weight of the beam.

**15.** On a common steelyard, a load of 1600 pounds is balanced by 360 pounds. Draw the diagrams of bending moment and shearing force in the steelyard bar, when thus loaded.

**16**. A countershaft, 6 feet long between centres of bearings, carries a driving pulley $A$, 20 inches diameter and distant 1 foot from the centre of the left-hand bearing, and a driven pulley $B$, 18 inches diameter and distant $3\frac{1}{2}$ feet from $A$. In both cases, the belt-drive is horizontal, being toward the front at $A$ and to the rear at $B$. The tension of the driving side at $A$ is 200 pounds; at $B$, 170 pounds. The weight at $A$ is 80 pounds; at $B$, 60 pounds. Find the maximum equivalent bending moment on the shaft.

**17**. In Example 14, let the cranks be offset, the neutral axis of each making an angle of 120 degrees with the centre line of the shaft. With this modification, draw the moment diagrams throughout, as in Fig. 20.

**18**. In a 4-cylinder, triple expansion engine, the high-pressure and intermediate-pressure cylinders are 32 and 52 inches, respectively, in diameter; the stroke is 48 inches; length of connecting rod, 96 inches; the H.P. and I.P. cranks are 90 degrees apart, the H.P. leading; the initial, unbalanced pressure on the H.P. piston is 200 pounds per square inch; on the I.P. piston, 50 pounds; the length of each crank pin is 19 inches; and the average length of the three bearings — one at each end of the section of the shaft and one between the two pairs of cranks — is 24 inches. Find the maximum bending moment on the H.P. and I.P. section of the shaft, due to the loads on the two pistons and the corresponding reactions at the bearings.

**19**. Assume three concentrated loads, $P_1$, $P_2$, and $P_3$, spaced at the constant distance $a$ between the first two loads and $b$ between the second and third. Let this series of loads cross a beam of span $s$ from left to right. Find the influence line for the left reaction.

**20**. Find the influence line for maximum shear for the series of loads, as above.

**21**. In Fig. 28, let there be two coupled locomotives crossing the plate girder bridge from left to right, the weights and spacing being those given in Example 15 and the locomotives being 9 feet apart. Find the maximum moments and shears.

N

**22.** Let the 5-panel truss, Fig. 25, have a span of 125 feet and a depth of 26 feet. Take the live load as the two coupled locomotives in Problem 21, followed at a distance of 5 feet by a train weighing 5000 pounds per lineal foot. Determine the maximum stresses in the members and find in which panel or panels counterbracing is necessary.

**23.** Let the Warren girder, Fig. 31, have a 'broken' lower chord, the depth at joints 6 and 8 being 6 feet; at joints 4 and 10, 5 feet; and at joints 2 and 12, 3 feet, 4 inches. Find the maximum live load stresses in the members, due to the load given in Example 16.

**24.** Find the centre of gravity of the deck beam whose dimensions are given in Example 20.

**25.** Find the centre of gravity of an angle with unequal legs, $7 \times 3\frac{1}{2}$ inches by $\frac{7}{16}$ inch thick.

**26.** Find the rectangular moment of inertia of the bulb angle whose dimensions are given in Example 19. Take the neutral axis as perpendicular to the web and use the method of the area of the equilibrium polygon.

**27.** The cross-section of a channel has the following dimensions: depth, 15 inches; area of section, 9.9 square inches; thickness of web, 0.4 inch; width of flange, 3.4 inches; thickness of flange, 0.9, tapering to 0.4 inch; distance of centre of gravity from outside of web, 0.794 inch.

Find the moment of inertia: (a) with neutral axis perpendicular to web at centre; (b) with neutral axis parallel with centre line of web.

**28.** A cast iron frame weighing 300 pounds is moved across a smooth iron floor plate by a force $P$ acting at an angle of 45 degrees with the horizontal. Take the coefficient of friction as 0.4 and find the magnitude of $P$.

**29.** The joint between the two sections of a vertical pump rod is made by a through key having one horizontal side and one side

tapered to an angle of $3^1$ degrees with the horizontal. In making the joint, the total stress to be produced in the rod is 1000 pounds. Using a coefficient of friction of 0.25, find the force which will be required to drive the key home.

**30.** The cover of a steam engine cylinder, $34^1$ inches in diameter is secured to the cylinder by thirty-four $1\frac{1}{4}$-inch steel studs. Considering friction, what power, applied to the nut by a wrench 2 feet long, will produce a total stress in each nut of 4500 pounds ?

**31.** Find the efficiency, at the mid-point of the forward stroke, of a double-acting, oscillating engine, disregarding the friction of the piston and piston rod. Data: Cylinder diameter, 15 inches; stroke, 18 inches; diameter of crank pin, $2\frac{1}{2}$ inches; of trunnion, 6 inches; unbalanced pressure on piston, 100 pounds per square inch; coefficient of friction, 0.185.

**32.** The shaft of a marine engine is transmitting 4000 horse power when revolving at a speed of 125 revolutions per minute, with the ship making 15 knots per hour. The thrust shaft has an external diameter of $13\frac{3}{4}$ inches. There are 11 thrust collars, each 2 inches wide, working in a cast iron bearing lined with white metal. Assume — as was formerly the practice — that the collar bearing is continuous throughout its circumference, and, with a coefficient of friction of 0.1, find the power expended in overcoming the friction of the thrust bearing.

**33.** Following the general methods given in Example 25, analyze the action of the Weston Differential Pulley Block or chain hoist. Take the coefficient of journal friction as 0.1, that of chain friction as 0.2, and the ratio between the diameters of the two sheaves in the upper block as $\frac{19}{8}$.

Show the relation between the power and the load and the reasons for the large loss by friction. Prove that the tackle is self-locking with regard to backward motion.

**34.** In Example 26, let the pitch diameters of gear wheels $A$, $B$, and $C$ be 6, 8, and 12 inches, respectively. Interpose a 10-inch

gear wheel between wheels $B$ and $C$. Taking the force $P$ as 75 pounds, the arm $L$ as 8 inches, the arm $L'$ as 15 inches, the coefficient of journal friction as 0.1, and that of tooth friction as 0.12, find the magnitude of the resistance $W$.

**35.** In Fig. 52, interpose a belt-tightening pulley to the left of the slack side $(T_1)$ of the belt and near the driving pulley $A$, thus altering the line of action of the slack side and increasing its arcs of contact. With this modification, construct the force polygons for the entire gear by the methods used previously.

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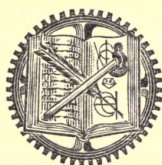
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2-month loans may be renewed by calling
(510) 642-6753
1-year loans may be recharged by bringing books
to NRLF
Renewals and recharges may be made 4 days
prior to due date

DUE AS STAMPED BELOW

OCT  6 1993

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